HEATH'S PRIMARY ARITHMETIC

D. C. HEATH & CO. BOSTON

HEATH'S

PRIMARY ARITHMETIC

A CLASS AT THE COUNTING TABLE.

Frontispiece.

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PRIMARY ARITHMETIC

BY

CHARLES E. WHITE

AND

BRUCE M. WATSON

SYRACUSE, N.Y.

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PREFACE

In education, as in other affairs of life, there is a tendency, on the part of many, to pursue good ideas to unreasonable extremes, and ofttimes to the exclusion of other ideas quite as good.

This book has been prepared with the design of bringing together the manifest advantages of the topically arranged text-book and the equally manifest advantages of the so-called "spiral" plan.

Each subject is treated by itself as exhaustively as the scope and purpose of the book will warrant. At the same time each new subject introduced is considered in all its relations with and bearing upon preceding subjects.

By the great abundance and variety of the drill work and problems throughout the book, all subjects are kept in constant review, every principle is applied in as many ways as possible, and the unity of the book is preserved.

The order of subjects is determined by the law of dependence, the degree of simplicity of the matter to be taught, and the relative importance of the respective subjects in the business of life.

The development of the various principles and processes has been written with great care and considerably in detail, with a view both to furnish the teacher a definite plan for presenting the work and to help the student in his efforts toward independent achievement.

Both the method and the matter of the book have been tested by actual use in the schoolroom; they are not in any sense an experiment.

Part I is a strictly primary arithmetic. The first few lessons are extremely simple, yet they furnish an illustration of the logical steps in the development of ideas of number. If any child begins the use of the book with elementary notions of number already developed in his mind to a certain point, the judicious teacher will be wise enough to begin where the child's previously acquired knowledge stops.

The development work preceding each table is designed to give the child a concrete understanding of the processes by which the table is made instead of forcing him to memorize abstract results obtained by making arbitrary combinations. But after a table has been thoroughly developed, the pupil

should be drilled in all its combinations until he can give results instantly without reference to the mental processes by which they may be obtained. To this end the drill charts should be used daily until all results can be given correctly without an instant's hesitation.

The problems following the tables were selected from lessons given by scores of successful primary teachers, and it is believed that they are far richer in variety of work and forms of statement than any list prepared by a single individual.

Part II is an introduction to written arithmetic proper. The color work, both here and in Part I, is introduced not merely to embellish the pages, but rather to furnish the best means of illustration and practice in certain arithmetical operations.

Much of the mental work in Part II may be used as supplementary to the questions in Part I.

Definitions are given only when and where they are needed.

In the treatment of fractions the fact that a fraction is an expression of division is kept prominent.

Throughout the book the authors have endeavored to insert whatever may help the pupil to an understanding of principles; to omit whatever is superfluous or may tend to confusion.

B. M. W.

SYRACUSE, February 8, 1901.

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PRIMARY ARITHMETIC

PART FIRST

To the Teacher. — For early work with numbers, a "counting table" is almost a necessity, whatever text-book may be used. It should be about two feet high, thirty inches wide, and six feet to fourteen feet in length, according to the size of the class. It should be surrounded by a slightly raised casing to prevent the counters from falling off.

Toothpicks and shoepegs are very good to use as counters, on account of their cheapness and convenience of handling.

1. Counters are in a pile in front of the pupils.

Take one counter.

Put another one with it. How many?

Put another with it. How many?

So on until ten have been taken.

Take three counters; seven counters; five; nine; six; four; ten; eight.

Count the girls in the class.

Count the boys in the class.

How many fingers have you on your right hand? On your left hand?

How many fingers have you on both hands?
How many fingers and thumbs have you?
Count ten with your eyes shut.

Give much drill on these numbers, using counters and other objects.

2. Make the figures 1, 2, 3, 4, 5, 6, 7, 8, 9.

Teacher place figure 1 on blackboard, and pupils take as many counters. Teacher place figure 2 on the blackboard and children take as many counters. So on with all the figures, one at a time, the pupils each time taking as many sticks as the figure means.

Teacher count out three, five, nine, seven, eight sticks, etc., and pupils make figures to represent them.

Make the figure to tell how many years old you are.
Make the figure to tell how many fingers you have.
Make the figure to tell how many hands and feet you have.
Make the figure to tell the hour for coming to school.

Much drill should be given in this kind of work.

How many leaves on this cherry branch?

How many cherries?

Make the figure that tells how many leaves.

Make the figure that tells how many cherries.

Make a stem holding 5 cherries; 8 cherries; 10 cherries.

How many roses in the picture?

How many rosebuds?

What figure tells how many roses?

What figure tells how many buds?

Count the roses and buds together.

How many parts has a woodbine leaf?

Make the figure.

How many leaves are there on a clover stem?

Make the figure.

Make two woodbine leaves.

Count the parts in the two leaves.

Count the red stripes in the flag.

Count the white stripes.

Make the figure that tells how many red stripes.

Make the figure that tells how many white stripes.

Count all the stripes.

Count the stars in the bottom row.

How many colors in the flag?

How many stripes below the blue field?

How many red stripes below the blue field?

Count the daisies in the picture.

How many?

What figure tells how many?

Count the tall daisies.

Make the figure that tells how many are not tall.

How many apples are in this picture?

Make the figure that tells how many apples.

Count the red apples.

Count the other apples.

Make the figure that tells how many red apples.

How many pears are there in the picture? Make the figure that tells how many.

One, 1. Two, 2. Three, 3. Four, 4.
Five, 5. Six, 6. Seven, 7.
Eight, 8. Nine, 9. Ten, 10.

3. Take ten sticks.

Tie them in a bunch, thus, 

Count out ten such bunches and tie them up.

Call each bunch "a ten."

Put one bunch on the table.

Put one stick beside it, thus, 

How many sticks are there? 11

Put another stick with it, thus, 

How many sticks are there? 12

Take another ten and put three with it, thus, 

How many are there? 13

Go on in this way to nineteen.

What does the figure 1 stand for in the number 12? In the number 13?

In 14, 15, 16, 17, 18, 19?

How many tens in 11? How many over?

In 12, 13, 14, 15, 16, 17, 18, 19?

4. Suppose we wish to write the number ten. We will use 1 to stand for ten the same as in the other numbers.

How many will there be over?

What shall we use to show that there is none over? 

We will use this figure, 0. Thus, 10

The figure, 0, is called naught, and is used where *nothing* belongs.

Take two tens. How many sticks in all? 
To write twenty we use 2 to stand for two tens, and 0 to show that there is none over, thus, 2 0

Take two tens and one, thus, 

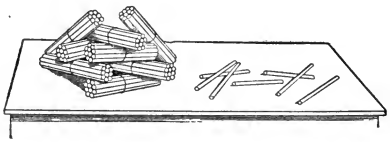
How many? Write it thus, 2 1

Make the figures for:  

5. In the same manner as in the preceding lesson, develop numbers to one hundred.

Write fifteen; twenty; eighteen; fourteen.
Write twenty-five; thirty-seven; forty.
Write eighty-two; ninety-six; seventy.
Write sixty-one; forty-seven; fifty.
Write seventy-nine; twenty-one; ninety.
Write forty-eight; nineteen; thirty-three.
Lay counters to make forty-two, thus, 
Lay counters to make sixty-three.
Lay counters to make seventeen.
Count the desks in your room.



What number will the counters on the table make?

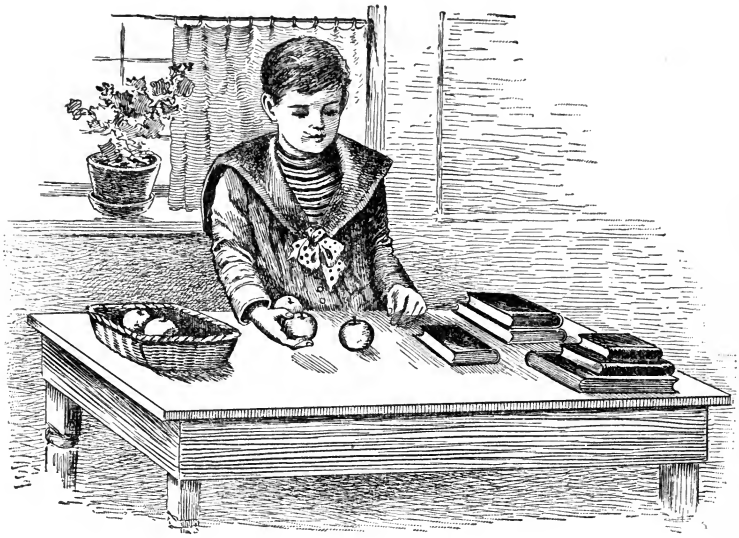
Count the hands in your schoolroom.
Write the number of books on the teacher's desk.
Count fifty forward and backward.
Read these numbers: 25, 21, 38, 45, 36, 28, 72, 64.
Write them in a column, so that the tens will be in a vertical line.
Write them in words.

ADDITION

6. Take one counter.
Take one more counter.
How many counters have you?
How did you get two counters?
One counter and one counter are how many counters?
One stick and one stick are how many sticks?
One book and one book are how many books?
One boy and one boy are how many boys?
One and one are how many?

● and ● are ?
One and one are ?
1 and 1 are ?

Write, 1 and 1 are 2.
Take two counters.
Put one more with them.
How many counters are there?
How did you get three counters?
Two counters and one counter are how many counters?



If the apples in the boy's hand be put with the apple on the table, how many will there be on the table? If the one book be put on the three books, how many books will there be in the pile?

Make a number story about the two books and one book.

Make a number story about the apples in the basket and the apple on the table. About one marble and one marble.

Two chairs and one chair are how many chairs?

Two and one are how many?

●● and ● are ?

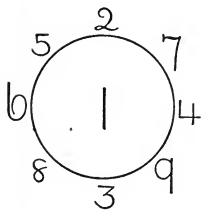
Two and one are ?

2 and 1 are ?

In the same way teach three and one, four and one, and so on to ten and one.

Have children write:

1 and 1 are 2	$1+1=2$
2 and 1 are 3	$2+1=3$
3 and 1 are 4	$3+1=4$
4 and 1 are 5	$4+1=5$
5 and 1 are 6	$5+1=6$
6 and 1 are 7	$6+1=7$
7 and 1 are 8	$7+1=8$
8 and 1 are 9	$8+1=9$
9 and 1 are 10	$9+1=10$
10 and 1 are 11	$10+1=11$



BLACKBOARD DRILL

Add the 1 to each of the other numbers around the circle.

7. Take 1 counter. Take two more counters.
How many counters have you?
How did you get three counters?
One counter and two counters are how many counters?
One apple and two apples are how many apples?
One and two are how many?
Take two counters. Take two more counters.
How many counters have you?

How did you get four counters?

Two counters and two counters are how many counters?

Two boards and two boards are how many boards?

Two sheep and two sheep are how many sheep?

Two and two are how many?

Take three counters. Take two more counters.

How many counters have you?

How did you get five counters?

Three counters and two counters are how many counters?

Three books and two books are how many books?

Three trees and two trees are how many trees?

Three and two are how many?

In a similar way teach the entire table of twos.
Have children write:

1 and 2 are 3	$1+2=3$
2 and 2 are 4	$2+2=4$
3 and 2 are 5	$3+2=5$
4 and 2 are 6	$4+2=6$
5 and 2 are 7	$5+2=7$
6 and 2 are 8	$6+2=8$
7 and 2 are 9	$7+2=9$
8 and 2 are 10	$8+2=10$
9 and 2 are 11	$9+2=11$
10 and 2 are 12	$10+2=12$

Make a number story about 3 dolls and 2 dolls. Another child give the answer.

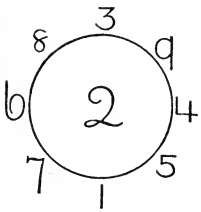
Make a number story about 7 red roses and 2 white roses. Give the answer.

Make a story about 6 and 2, 7 and 2, etc.

● ● ● ● ● ● and ● ● are ?

Six and two are ?

6 + 2 = ?



BLACKBOARD DRILL

Add the 2 to each of the other numbers quickly.

8. Take 1 counter. Take 3 more counters.

How many counters have you?

How did you get 4 counters?

One counter and three counters are how many counters?

One stick and three sticks are how many sticks?

One marble and three marbles are how many marbles?

One cat and three cats are how many cats?

One and three are how many?

Take two counters. Take three more counters.

How many counters have you?

How did you get five counters?

Two counters and three counters are how many counters?

● ● and ● ● ● are how many dots?

Two cows and three cows are how many cows?

Two and three are how many?

In a similar way, teach all the table of threes.

Make a number story about 2 red cherries and 3 red cherries. Another child give the answer.

Make a story about 8 green leaves and 2 green leaves.

Make a number story about 2 boys and 5 boys.

Make a number story about 7 dollars and 3 dollars. About 9 and 3.

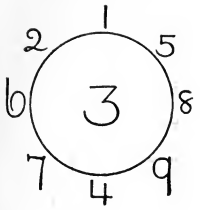
Have children write:

1 and 3 are 4	$1+3=4$
2 and 3 are 5	$2+3=5$
3 and 3 are 6	$3+3=6$
4 and 3 are 7	$4+3=7$
5 and 3 are 8	$5+3=8$
6 and 3 are 9	$6+3=9$
7 and 3 are 10	$7+3=10$
8 and 3 are 11	$8+3=11$
9 and 3 are 12	$9+3=12$
10 and 3 are 13	$10+3=13$

In a similar manner teach the entire table of Addition, giving much drill with both concrete and abstract work, as you proceed, to fix the tables in the minds of the children.

Vary the work, sometimes the teacher doing the work and pupil telling what she did, and the result.

Have children make questions.



BLACKBOARD DRILL

Add the 3 to each of the other numbers quickly.

9. TABLE OF ADDITION

1+1= 2	1+2= 3	1+3= 4	1+4= 5	1+ 5= 6
2+1= 3	2+2= 4	2+3= 5	2+4= 6	2+ 5= 7
3+1= 4	3+2= 5	3+3= 6	3+4= 7	3+ 5= 8
4+1= 5	4+2= 6	4+3= 7	4+4= 8	4+ 5= 9
5+1= 6	5+2= 7	5+3= 8	5+4= 9	5+ 5=10
6+1= 7	6+2= 8	6+3= 9	6+4=10	6+ 5=11
7+1= 8	7+2= 9	7+3=10	7+4=11	7+ 5=12
8+1= 9	8+2=10	8+3=11	8+4=12	8+ 5=13
9+1=10	9+2=11	9+3=12	9+4=13	9+ 5=14
10+1=11	10+2=12	10+3=13	10+4=14	10+ 5=15

1+6= 7	1+7= 8	1+8= 9	1+9=10	1+10=11
2+6= 8	2+7= 9	2+8=10	2+9=11	2+10=12
3+6= 9	3+7=10	3+8=11	3+9=12	3+10=13
4+6=10	4+7=11	4+8=12	4+9=13	4+10=14
5+6=11	5+7=12	5+8=13	5+9=14	5+10=15
6+6=12	6+7=13	6+8=14	6+9=15	6+10=16
7+6=13	7+7=14	7+8=15	7+9=16	7+10=17
8+6=14	8+7=15	8+8=16	8+9=17	8+10=18
9+6=15	9+7=16	9+8=17	9+9=18	9+10=19
10+6=16	10+7=17	10+8=18	10+9=19	10+10=20

Give three answers to each question below:

1. ? + ? = 10	9. ? + ? = 12
2. ? + ? = 17	10. ? + ? = 15
3. ? + ? = 9	11. ? + ? = 21
4. ? + ? = 20	12. ? + ? = 11
5. ? + ? = 14	13. ? + ? = 8
6. ? + ? = 7	14. ? + ? = 6
7. ? + ? = 16	15. ? + ? = 18
8. ? + ? = 13	16. ? + ? = 19

Addition Drill Chart

This chart contains all additions which result in sums no larger than 20. It should be copied on the blackboard, and

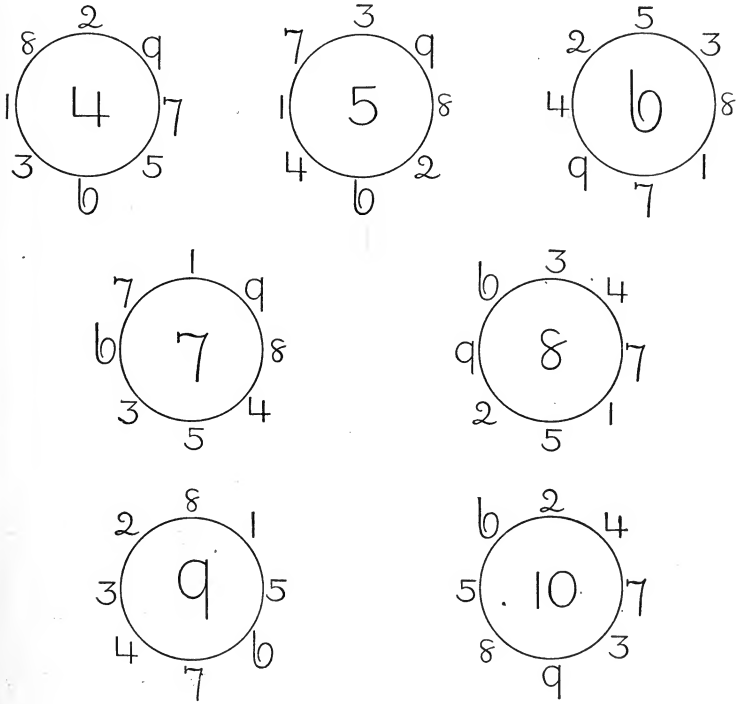
6	5	6	10	2	8	3	3	2	7	7
7	1	5	4	2	4	1	9	1	5	7
2	4	5	10	4	3	6	7	10	5	2
5	7	3	2	4	4	4	3	1	5	6
8	6	2	3	2	1	10	4	6	7	8
3	1	9	3	8	1	3	1	6	1	9
9	4	6	9	2	10	8	6	9	10	3
1	5	3	5	7	8	1	8	6	5	2
10	2	10	8	9	4	10	7	8	7	10
9	4	10	5	9	9	6	9	8	8	7

children should recite the sums every day until they can do so without making an error. Vary the drill. Begin at different

places and go through the entire chart to the place of beginning. Change the order, going sometimes to right, and to left, and sometimes up or down. Vary concert and individual recitation with and without the pointer. Sometimes pupils use the pointer.

Use the chart persistently.

10. To the Teacher. Pupils need drill on this kind of work till they can give results instantly.



1. How many boys are 5 boys and 3 boys?
2. Three eggs and 4 eggs are how many eggs?

3. How many are six tops and 3 tops?

4. How many are 8 hens and 7 hens?

5. Two cats and four cats are how many cats?

6. Five books and two books are how many books?

7. Four hats and six hats are how many hats?

8. Eight slates and nine slates are how many?

9. 3 chairs and 5 chairs are how many chairs?

10. Six balls and three balls are how many balls?

11. Five and two are how many?

12. Nine and three are how many?

13. How many are four and four?

14. May had three cents, and I gave her 8 cents; how many cents did she have then?

15. John found 4 eggs in one nest and six in another; how many eggs did he find?

16. James had 7 cents and found 4 more; how many cents did he then have?

17. Make a number story about 3 birds and 6 birds. 4 men and 6 men. 5 and 2.

11. 1. A boy saw 6 squirrels in one tree and 4 in another. How many squirrels did he see?

2. 5 oranges + 3 oranges = ? 7 pins + 9 pins = ? 6 + 7 = ?

3. Henry had 5 cents, John 4, and George 8. How many cents did they all have?

4. A wagon carried 7 women, 5 men, and 3 children. How many persons did it carry?

5. $3+2+4=?$	22. $8+5=?$
6. $2+7+5=?$	23. $3+8+1+1=?$
7. $4+8+2=?$	24. $4+6+9=?$
8. $6+5+3=?$	25. $2+3+6+4=?$
9. $9+2+5=?$	26. $8+2+2+6=?$
10. $6+1+4=?$	27. $18+3=?$
11. $6+1+5=?$	28. $28+3=?$
12. $5+6+1=?$	29. $48+3=?$
13. $7+2+8=?$	30. $69+5=?$
14. $1+2+3+9=?$	31. $49+10+1=?$
15. $6+3+1+8=?$	32. $95+5=?$
16. $1+2+4+8=?$	33. $7+8+5=?$
17. $1+9+2+1=?$	34. Add by fives from 0 to 20.
18. $5+2+3+9=?$	35. Add by threes from 0 to 30.
19. $6+4+8=?$	
20. $6+3+4=?$	
21. $6+1+6+2=?$	

36. Count by 2's from 0 to 40. From 1 to 21.
37. Count by 4's from 0 to 40. From 2 to 30.
38. Count by 6's from 0 to 48. From 5 to 42.
39. Count by 7's from 0 to 35. From 5 to 47.
40. Count by 8's from 0 to 48. From 6 to 54.
41. Make a number story about 5 swallows, 9 swallows, and 3 swallows.
42. Make a number story about 5, 3, and 8.

12. 1. Eight boys and 5 boys are how many boys?
2. Seven eggs and three eggs are how many eggs?

3. 9 tops and three tops are how many tops?
4. 6 cats and 2 cats are how many cats?
5. 7 books and 5 books are how many books?
6. 10 hats and 4 hats are how many hats?
7. 10 slates and 1 slate are how many slates?
8. How many chairs are 8 chairs and 3 chairs?
9. How many balls are 10 balls and 6 balls?
10. How many dogs are 6 dogs and 5 dogs?
11. 11 days and 4 days are how many days?
12. Seven apples and 4 apples are how many apples?
13. 11 balls and 5 balls are how many balls?
14. 8 cows and 3 cows are how many cows?
15. 11 and 5 are how many?
16. 10 and four are how many?
17. How many are 11 and 6?
18. John spent 3 cents for candy, 5 cents for a top, and 6 cents for marbles. How much did he spend in all?
19. A man had three cows in one field, four in another, and seven in another. How many did he have in all? Make a picture of the fields using dots for cows.

20. $9+4=?$	28. $5+7=?$	36. $7+9=?$
21. $8+5=?$	29. $4+8=?$	37. $8+10=?$
22. $6+2=?$	30. $4+9=?$	38. $2+5+3=?$
23. $9+2=?$	31. $5+10=?$	39. $4+6+7=?$
24. $6+2=?$	32. $8+5=?$	40. $9+1+5=?$
25. $6+3=?$	33. $9+6=?$	41. $8+2+3=?$
26. $9+4=?$	34. $8+7=?$	42. $9+9+2+1=?$
27. $5+5=?$	35. $8+8=?$	43. $1+2+3+4+5=?$

13. 1. George picked from one tree,  from another tree, and from another tree. How many did he pick in all?

2. There were  in one row, 4 in another, and 9 in another. How many trees were there in all?

3. A girl earned 8 cents, found 5 cents, and had 3 cents given to her. How many cents had she?

4. A cat caught  in the house, in the barn, and  in the field. How many did she catch in all?

5. A gardener gave Ned  , Nell  , and Will as many as the other two. How many 's did he give all?

6. Frank ate three plums, gave Charles nine, and had four left. How many plums had he at first?

7. I saw on the lawn 10 sparrows, 6 robins, and 4 orioles. How many birds did I see in all?

8. A boy paid 2 dollars for shoes, 1 dollar for a hat, 5 dollars for a coat, and 9 dollars for books. How much did he pay for all?

9. A boy caught 10 fish, and his sister 9. How many fish did both catch?

10. Nancy walked 2 miles on Monday, 3 miles on Wednesday, 4 miles on Thursday, and 9 miles on Saturday. How far did she walk?

11. $1+2+3+4+5=?$	16. $7+7+6=?$
12. $2+1+9=?$	17. $12+8=?$
13. $10+10=?$	18. $5+1+6+3=?$
14. $4+8+2=?$	19. $3+4+5=?$
15. $9+1+6=?$	20. $6+9=?$

SUBTRACTION

14. Teacher take two counters, and children tell how many. Teacher take one away, and children tell what she has done, and how many are left.

Children take two counters, take one away, and tell how many are left.

Two counters less one counter are how many counters?

Two boxes less one box are how many boxes?

Two knives less one knife?

Two flowers less one flower?

Two less one are how many?

Teacher take three counters, and children tell how many. Teacher take one away, and children tell what she did, and number left.

Three counters less one counter are how many counters?

Three books less one book are how many books?

Three less one are how many?

 less  are ?

Three less one are ?

3 less 1 are ?

So on till all the ones are taught.
Children write:

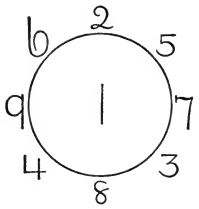
2 less 1 are 1	2 − 1 = 1
3 less 1 are 2	3 − 1 = 2
4 less 1 are 3	4 − 1 = 3
5 less 1 are 4	5 − 1 = 4
6 less 1 are 5	6 − 1 = 5
7 less 1 are 6	7 − 1 = 6
8 less 1 are 7	8 − 1 = 7
9 less 1 are 8	9 − 1 = 8
10 less 1 are 9	10 − 1 = 9
11 less 1 are 10	11 − 1 = 10

Make a number story about the holly leaves. Give the answer.

Make an adding story about the holly buds. Give the answer.

Make a number story about a tree, 9 blackbirds, and 1 blackbird.

Make a number story about 8 and 1.



BLACKBOARD WORK

Take one from each of the other numbers.

15. Teacher take three counters, and children tell how many.

Take away two counters. Children tell what has been done and how many are left.

Children take three counters. Take away two counters. Tell what has been done, and how many are left.

Three counters less two counters are how many counters?

Three peaches less two peaches are how many peaches?

Three less two are how many?

Teacher take four counters. Take away two counters. How many are left? Children tell what has been done, and the result.

Teacher take five counters; take away two. Children watch carefully and tell the whole story — what has been done and the result.

 less  are ?

Five less two are ?

5 − 2 = ?

How many oranges on the plate in the picture?

If the girl takes away 2 oranges, how many will be left?

Five less two are how many?

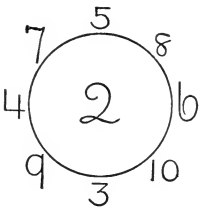
So on through the remainder of table of twos.

Children write:

3 less 2 are 1	$3 - 2 = 1$
4 less 2 are 2	$4 - 2 = 2$
5 less 2 are 3	$5 - 2 = 3$
6 less 2 are 4	$6 - 2 = 4$
7 less 2 are 5	$7 - 2 = 5$
8 less 2 are 6	$8 - 2 = 6$
9 less 2 are 7	$9 - 2 = 7$
10 less 2 are 8	$10 - 2 = 8$
11 less 2 are 9	$11 - 2 = 9$
12 less 2 are 10	$12 - 2 = 10$

Make a number story about a pond, a bank, 9 ducks, and 2 ducks. Give the answer.

Make a number story about 8 less 2.



Blackboard Work

Take 2 from each of the other numbers.

16. Children watch.

Teacher take four counters, and take away three.

Children tell what is done, and the result.

Four counters less three counters are how many counters?

Four roses less three roses?

Four hats less three hats?

Four less three?

Children watch.

Teacher take five counters, and take away three.

Children do just what teacher has done, tell what they have done, and the result.

Five less three are how many?

So on until all the threes have been taught.

Make a number story about 9 eggs, a nest, and 3 eggs. Give the answer.

Make a story about 12 less 3.

Children write the table of threes as in previous lessons.

1. Mary had 9 roses, and gave Helen 3. How many roses had she left?

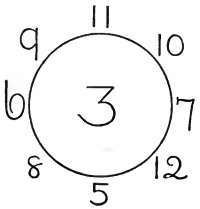
NOTE.—If a child fails, let him take 9 counters, call them roses; take away three "roses," and see how many roses are left.

2. George earned 5 cents, and spent 3 cents. How many cents had he left?

3. Seven quarts of water were in a pail. Three quarts leaked out. How many were left?

4. Henry has 7 cents. How much must he earn to buy a ball for 10 cents?

5. Make a question that has 3 for an answer.



BLACKBOARD DRILL

Subtract 3 from each of the other numbers around the circle.

In a similar way teach all the table of subtraction. Show with counters that $2-2=0$, $3-3=0$, $4-4=0$, $5-5=0$.

17. TABLE OF SUBTRACTION

1−1= 0	2−2= 0	3−3= 0	4−4= 0	5−5= 0
2−1= 1	3−2= 1	4−3= 1	5−4= 1	6−5= 1
3−1= 2	4−2= 2	5−3= 2	6−4= 2	7−5= 2
4−1= 3	5−2= 3	6−3= 3	7−4= 3	8−5= 3
5−1= 4	6−2= 4	7−3= 4	8−4= 4	9−5= 4
6−1= 5	7−2= 5	8−3= 5	9−4= 5	10−5= 5
7−1= 6	8−2= 6	9−3= 6	10−4= 6	11−5= 6
8−1= 7	9−2= 7	10−3= 7	11−4= 7	12−5= 7
9−1= 8	10−2= 8	11−3= 8	12−4= 8	13−5= 8
10−1= 9	11−2= 9	12−3= 9	13−4= 9	14−5= 9
11−1=10	12−2=10	13−3=10	14−4=10	15−5=10

TABLE OF SUBTRACTION (*Continued*)

6−6= 0	7−7= 0	8−8= 0	9−9= 0	10−10= 0
7−6= 1	8−7= 1	9−8= 1	10−9= 1	11−10= 1
8−6= 2	9−7= 2	10−8= 2	11−9= 2	12−10= 2
9−6= 3	10−7= 3	11−8= 3	12−9= 3	13−10= 3
10−6= 4	11−7= 4	12−8= 4	13−9= 4	14−10= 4
11−6= 5	12−7= 5	13−8= 5	14−9= 5	15−10= 5
12−6= 6	13−7= 6	14−8= 6	15−9= 6	16−10= 6
13−6= 7	14−7= 7	15−8= 7	16−9= 7	17−10= 7
14−6= 8	15−7= 8	16−8= 8	17−9= 8	18−10= 8
15−6= 9	16−7= 9	17−8= 9	18−9= 9	19−10= 9
16−6=10	17−7=10	18−8=10	19−9=10	20−10=10

Give three answers to each of these questions:

1. 20 − ? = ?	7. 14 − ? = ?	13. 8 − ? = ?
2. 19 − ? = ?	8. 13 − ? = ?	14. 7 − ? = ?
3. 18 − ? = ?	9. 12 − ? = ?	15. 6 − ? = ?
4. 17 − ? = ?	10. 11 − ? = ?	16. 5 − ? = ?
5. 16 − ? = ?	11. 10 − ? = ?	17. 4 − ? = ?
6. 15 − ? = ?	12. 9 − ? = ?	18. 3 − ? = ?

SUBTRACTION DRILL CHART

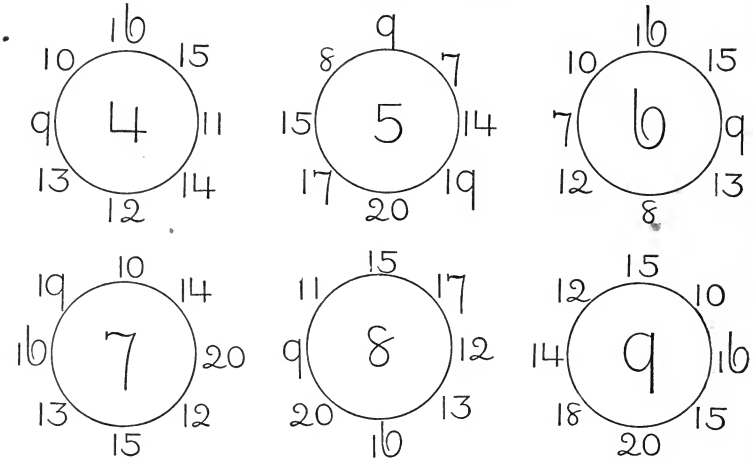
This chart contains all the subtraction table. It should be copied on the blackboard, and remainders recited by the class every day till mastered. Vary the drill. Begin sometimes at the upper left-hand corner, sometimes at the lower right-hand corner. Go up and down, to the right and the left. Recite sometimes in concert and sometimes individually, teacher or pupil using the pointer, and sometimes without the pointer.

11	4	15	9	13	12	6	15	11	16
−10	−2	−7	−9	−8	−9	−5	−6	−2	−8
9	12	13	6	11	10	19	7	6	8
−6	−5	−9	−1	−6	−8	−10	−4	−2	−3
8	11	7	8	9	6	11	3	3	15
−5	−5	−3	−6	−3	−3	−5	−2	−1	−8
10	13	9	14	13	9	11	10	9	5
−6	−5	−7	−6	−7	−1	−8	−7	−8	−3
14	12	11	17	6	7	2	14	8	17
−10	−6	−3	−9	−4	−6	−2	−8	−5	−10
12	10	12	12	11	7	9	5	8	17
−3	−1	−7	−8	−9	−1	−5	−4	−8	−8
9	7	8	8	4	16	18	9	4	8
−5	−2	−2	−7	−3	−10	−10	−2	−1	−1
13	15	5	10	11	7	3	13	5	16
−6	−9	−2	−9	−1	−5	−3	−5	−1	−9
14	5	7	14	12	10	12	11	16	10
−5	−5	−7	−9	−5	−5	−10	−7	−7	−5
10	4	10	6	14	18	15	2	13	1
−10	−4	−3	−6	−7	−9	−10	−1	−10	−1

For the Teacher: — After a table has once been developed, the pupil should be thoroughly drilled upon it.

Give much concrete work. In drill work, do not allow the child to use counters in finding the answers, but require him to rely upon his memory. If he gives a wrong answer, he should find and correct his error by means of the counters.

BLACKBOARD DRILL



18. 1. May had 4 pictures and gave away 2 of them. How many had she left?

2. In a field were 11 sheep. The dogs killed 5 of them. How many were left?

3. In a class were 9 scholars. Four of them were boys. How many were girls?

4. Henry had 13 marbles. He lost 3 and gave away 1. How many had he left?

5. In a class were 12 scholars. Six of them were girls. How many were boys?

6. The number of boys in a class was ten. The number of girls was 3 less. How many girls were there?

7. William is 14 years old. James is nine. William is how much older than James?

8. Celia is 12 years old. Sarah is 5 years younger. How old is Sarah?

9. A farmer planted 18 trees. Ten of them died. How many lived?

10. A hen hatched 11 chickens. Two of them were white and the rest speckled. How many of them were speckled?

11. 13 less 6 are ?	24. 19 − 9 = ?
12. 12 less 5 are ?	25. 17 − 8 = ?
13. 9 less 5 are ?	26. 12 − 7 = ?
14. 8 less 6 are ?	27. 12 − 6 = ?
15. 5 less 2 are ?	28. 14 − 7 = ?
16. 10 less 9 are ?	29. 18 − 8 = ?
17. 11 less 4 are ?	30. 17 − 8 = ?
18. 13 less 4 are ?	31. 18 − 9 = ?
19. 15 less 5 are ?	32. 11 − 5 = ?
20. 14 less 7 are ?	33. 12 − 4 = ?
21. 9 less 5 are ?	34. 15 − 8 = ?
22. 8 less 6 are ?	35. 20 − 10 = ?
23. 9 less 5 are ?	36. 19 − 9 = ?

19. 1. A boy had 5 cents and earned 7 cents more. How many cents did he then have?

2. A boy had 15 cents and spent 10 cents. How many cents had he left?

3. A lady had 7 pies on a shelf and 3 on a table. How many pies had she?

4. A man had 8 horses and bought 6 more. How many had he then?

5. There were 17 apples in a dish. A boy took out 9 of them. How many were left?

6. I had 9 dollars. I paid 5 dollars for a chair and the rest of my money for a table. What did the table cost?

7. There were 8 boys in a chestnut tree and 7 on the ground. How many in all?

8. Henry had 5 books. His aunt gave him 3 and his sister 4. How many had he then?

9. Mary is 15 years old and John is 5 years younger. How old is John?

10. Make a number story about 6 and 5.

11. Make a number story about 13 and 4.

12. Make a number story about 2 and 7.

13. Make a subtracting story about hens.

14. Make an adding story about birds.

20. 1. A clerk cut 10 yards of cloth from a piece containing 19 yards. How many yards were left?

2. Seven ducks came to a pond; then 8 more came; then 6 flew away. How many were left?

3. Robert earned 6 cents on Monday, 4 cents on Tuesday, and 7 cents on Wednesday. He spent 9 cents on Thursday. How much had he left?

4. There were 15 words in the spelling lesson. Richard missed 4. How many did he spell right?

5. A carpenter cut 7 feet from a 16-foot board. How long a piece was left?

6. In a basket were 5 red plums, 5 green ones, and 9 blue ones. How many were there?

7. A boy had a nickel and a dime. He bought a top for 3 cents and some buns for 4 cents. How much money had he left?

8. A farmer sold some wheat for 10 dollars and some potatoes for 7 dollars. He wanted to buy some clothing for 20 dollars. How much more money did he need?

9. A boy had 13 cents. He spent 6 cents and earned 9 cents. How much had he then?

10. Count by threes from 2 to 23.

11. $8 + 7 = ?$	18. $14 - 5 = ?$
12. $15 - 8 = ?$	19. $14 - 9 = ?$
13. $7 = 15 - ?$	20. $9 + ? = 14$
14. $18 = 9 + ?$	21. $2 + 4 + 7 - 8 + 3 = ?$
15. $9 + ? = 18$	22. $13 - 4 - 6 + 3 + 8 = ?$
16. $9 + 9 = ?$	23. $1 + 2 + 3 - 3 + 7 + 9 = ?$
17. $14 = 5 + ?$	24. $12 + 1 + 1 + 1 - 10 + 4 = ?$

Count by 2's backward from 28 to 0. From 39 to 1.

Count by 5's from 0 to 50, and back again.

Add by 6's from 0 to 54, and subtract by 6's back to 0.

MULTIPLICATION

21. Take one counter.

Take one more counter.

What have you done?

How many times did you take one counter?

Two times 1 counter are how many counters?

Two times 1 stick are how many sticks?

Two times 1 boy are how many boys?

Two times 1 chair are how many chairs?

Two times 1 are how many?

Take one counter. Take one more counter. Take one more counter. How many counters have you?

What did you do to get 3 counters?

Three times 1 counter are how many counters?

Three times 1 pencil are how many pencils?

Three times 1 day are how many days?

Take 1 counter. Take 1 more counter. Take 1 more counter. Take 1 more counter. What did you do? How many counters have you?

Four times 1 counter are how many counters?

Four times 1 star are how many stars?

Four times 1 leaf are how many leaves?

Four times 1 are how many?

Similarly teach to "10 times 1 are 10."

Have children write:

2 times 1 are 2	$1 \times 2 = 2$
3 times 1 are 3	$1 \times 3 = 3$
4 times 1 are 4	$1 \times 4 = 4$
5 times 1 are 5	$1 \times 5 = 5$
6 times 1 are 6	$1 \times 6 = 6$
7 times 1 are 7	$1 \times 7 = 7$
8 times 1 are 8	$1 \times 8 = 8$
9 times 1 are 9	$1 \times 9 = 9$
10 times 1 are 10	$1 \times 10 = 10$

22. Lay 2 counters. Lay 2 more counters. Thus,

How many counters have you?
What did you do to get four counters?
How many times did you take 2 counters?
Two times two counters are how many counters?
Two times 2 dots are how many dots?
Two times 2 balls are how many balls?
Two times 2 are how many?
Lay 2 counters. Lay 2 more. Lay 2 more. Thus,

How many counters have you?
How did you get six counters?
Three times 2 counters are how many counters?
Three times 2 pegs are how many pegs?
Three times 2 knives are how many knives?
Three times 2 are how many?
Lay 2 counters, 2 more, 2 more, 2 more. Thus,

How many counters have you?
How did you get eight counters?
Four times 2 counters are how many counters?
Four times 2 bats are how many bats?
Four times 2 rabbits are how many rabbits?
Four times 2 are how many?

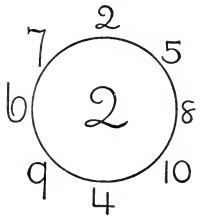
Similarly teach the rest of this table.

Children write:

2 times 2 are 4	$2 \times 2 = 4$
3 times 2 are 6	$2 \times 3 = 6$
4 times 2 are 8	$2 \times 4 = 8$
5 times 2 are 10	$2 \times 5 = 10$
6 times 2 are 12	$2 \times 6 = 12$
7 times 2 are 14	$2 \times 7 = 14$
8 times 2 are 16	$2 \times 8 = 16$
9 times 2 are 18	$2 \times 9 = 18$
10 times 2 are 20	$2 \times 10 = 20$

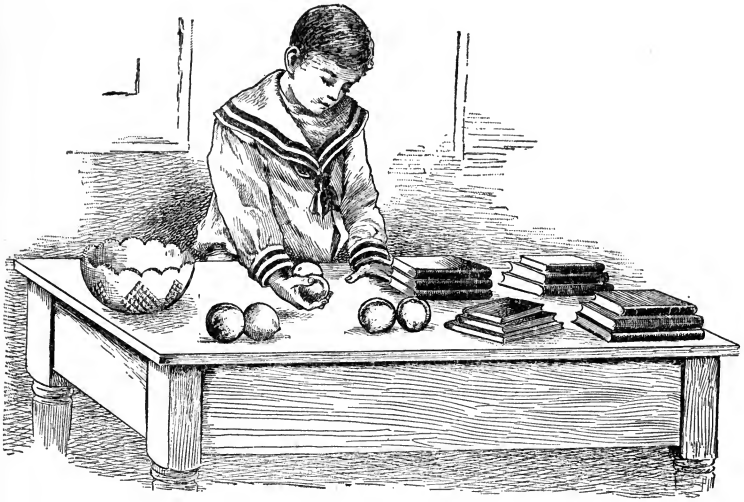
23. 1. In our yard are 3 trees, and 2 birds live in each tree. How many birds live in all the trees?

2. How many hands have 4 boys?
3. How many thumbs have 5 girls?
4. How many shoes in 10 pair?
5. How many sleeves are there in 7 coats?
6. How many noses have 6 dogs?
7. How many eyes have 6 sheep?
8. Make a story about 3 pockets and 2 cents.
9. Make a story about 6 times 2 feet.
10. Make a story about 8 times 2 gloves.
11. Make a story about 9 windows and 2 panes of glass.



BLACKBOARD DRILL

Take 2 times each number around the circle.



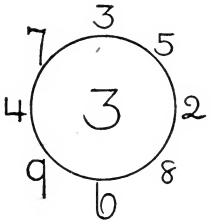
How many peaches do you see in the picture? If the boy puts all of the peaches into the dish, taking two at a time, how many times will it take him?

Make a number story about the books on the table.

24. Teacher lay 3 counters, and 3 more counters. Children watch and tell the teacher what she did and how many counters she has.

Two times 3 counters are how many counters?
Two times 3 pegs are how many pegs?
Two times 3 sticks are how many sticks?
Two times 3 trees are how many trees?
Two times 3 are how many?

In the same way teach the rest of this table to "10 times 3." Children write the table as in preceding lesson.



BLACKBOARD DRILL

Find 3 times each of the numbers around the circle.

1. Two girls had 3 cents apiece. How many cents had both?

2. Six men earn three dollars a day apiece. How many dollars do all earn in a day?

3. Fred has 9 three-cent pieces. How much money has he?

4. There are 3 feet in 1 yard. How many feet in 7 yards?

5. If a cord of wood costs 3 dollars, what will 8 cords cost?

6. What is the cost of 4 pencils at 3 cents apiece?

7. How far can you walk in 8 hours, if you walk 3 miles every hour?

8. If it takes 3 horses to draw a van, how many horses can draw 8 vans?

9. If lemons cost 4 cents apiece, what will 3 lemons cost?

10. In an orchard there are 3 rows of trees with 10 trees in a row. How many trees are in the orchard?

In the same way teach the remainder of the multiplication table. Give much concrete work after each part of the table.

Call attention to $1 \times 0 = 0$, $2 \times 0 = 0$, $3 \times 0 = 0$, etc.

Also, $1 \times 1 = 1$, $1 \times 2 = 2$, $1 \times 3 = 3$, etc.

25. TABLE OF MULTIPLICATION

1× 1= 1	2× 1= 2	3× 1= 3	4× 1= 4	5× 1= 5
1× 2= 2	2× 2= 4	3× 2= 6	4× 2= 8	5× 2= 10
1× 3= 3	2× 3= 6	3× 3= 9	4× 3=12	5× 3= 15
1× 4= 4	2× 4= 8	3× 4=12	4× 4=16	5× 4= 20
1× 5= 5	2× 5=10	3× 5=15	4× 5=20	5× 5= 25
1× 6= 6	2× 6=12	3× 6=18	4× 6=24	5× 6= 30
1× 7= 7	2× 7=14	3× 7=21	4× 7=28	5× 7= 35
1× 8= 8	2× 8=16	3× 8=24	4× 8=32	5× 8= 40
1× 9= 9	2× 9=18	3× 9=27	4× 9=36	5× 9= 45
1×10=10	2×10=20	3×10=30	4×10=40	5×10= 50
6× 1= 6	7× 1= 7	8× 1= 8	9× 1= 9	10× 1= 10
6× 2=12	7× 2=14	8× 2=16	9× 2=18	10× 2= 20
6× 3=18	7× 3=21	8× 3=24	9× 3=27	10× 3= 30
6× 4=24	7× 4=28	8× 4=32	9× 4=36	10× 4= 40
6× 5=30	7× 5=35	8× 5=40	9× 5=45	10× 5= 50
6× 6=36	7× 6=42	8× 6=48	9× 6=54	10× 6= 60
6× 7=42	7× 7=49	8× 7=56	9× 7=63	10× 7= 70
6× 8=48	7× 8=56	8× 8=64	9× 8=72	10× 8= 80
6× 9=54	7× 9=63	8× 9=72	9× 9=81	10× 9= 90
6×10=60	7×10=70	8×10=80	9×10=90	10×10=100

1. ?×?=81
2. ?×?=12 (Two answers)
3. ?×?=24 (Two answers)
4. ?×?=32
5. ?×?=20 (Two answers)
6. ?×?=16 (Two answers)
7. ?×?=18 (Two answers)
8. ?×?=25
9. ?×?=45
10. ?×?=56
11. ?×?=63
12. ?×?=90
13. ?×?=72
14. ?×?=50
15. ?×?=21
16. ?×?=14
17. ?×?=35
18. ?×?=64
19. ?×?=40
20. ?×?=28

Multiplication Drill Chart

This chart contains all combinations of two factors from 1 to 10. It should be copied on the blackboard, and the products should be recited daily till mastered. Vary the drill as in Subtraction and Addition. Persist in the use of the chart.

9	5	9	3	6	7	2	7	8	9	10
3	4	7	3	4	6	2	5	1	5	2
10	7	6	3	2	4	10	3	10	8	8
7	1	5	2	1	3	5	1	8	2	8
8	8	1	5	7	10	6	10	6	4	9
3	6	1	5	2	3	2	4	3	1	2
9	6	9	9	8	5	8	7	10	9	10
8	1	9	1	7	1	5	3	6	4	9
9	8	5	6	10	7	7	4	10	5	4
6	4	2	6	1	7	4	2	10	3	4

Occasionally let one pupil read the chart aloud from the book, using the upper number for a multiplier, the other pupils of the class giving the products.

26. Drill on the table of multiplication.

If a pupil gives a wrong answer, let him find and correct his mistake by means of counters.

1. What will 10 oranges cost at 5 cents apiece? 7 lemons at 3 cents? 6 tops at 8 cents? 9 sleds at 4 dollars?

2. Six boys each picked 7 quarts of berries. How many quarts did all pick?

3. A newsboy made 5 cents a day for 9 days. How much did he make?

4. How much could a newsboy make in 5 days, at 9 cents a day? In 8 days, at 10 cents a day? In 3 days, at 7 cents a day?

5. A dime is 10 cents. How many cents in 2 dimes? In 4 dimes? In 5 dimes? In 7 dimes? In 9 dimes?

6. Which is greater, 9 times 8, or 8 times 9?

7. A man has 4 fields of corn, and 4 men work in each field. How many men in all the fields?

8. A lady has 5 boxes, with 6 spoons in each box. How many spoons has she?

9. Write the table of 8's, 9's, 7's, 5's, 6's, 4's.

10. At 10 cents a dozen, what will 8 dozen bananas cost?

11. How many wings have 5 birds?

12. Rose has 8 marbles, and her brother 7 times as many. How many has her brother?

13. If there are 9 plants each in 5 rows, how many plants in all?

14. If a dozen eggs cost 12 cents, what will 10 dozen cost?

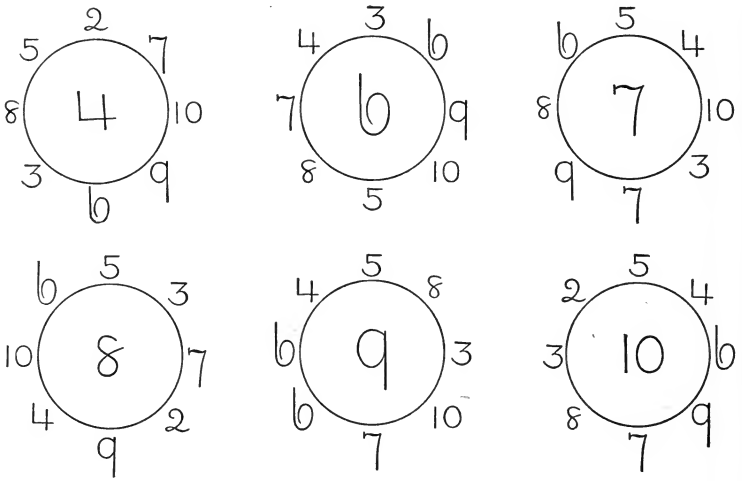
15. Harry can pick 4 quarts of berries in one day. At the same rate, how many can he pick in 10 days?

Make stories about this picture.

Make a picture of 5 squares in 6 rows, and tell a number story about it.

Blackboard Drill

Multiply the figure in the centre of each circle by the different numbers around it. Answers should be instantaneous.

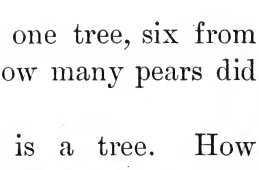


27. Addition, Subtraction, and Multiplication.

1. A man saw 4 ducks in one pond, 5 in another, and 7 in another. How many ducks did he see? Make a picture.

2. A man picked six pears from one tree, six from another, and five from another. How many pears did he pick?

3. Suppose each of these dots is a tree. How many rows of trees are there? How many trees in a row? How many trees in the orchard? How do you find it?



4. Make an orchard of 3 rows, with 9 trees in a row. How many trees in the orchard?

5. Make an orchard of 4 rows, with 7 trees in a row. How many trees in the orchard?

6. Nelly had 11 apples and gave away 5. How many had she left?

7. Make a story about 3 apples, 4 apples, and 7 apples.

8. Will's father gave him 20 cents. He paid five cents for a tablet and 5 cents for a pencil. How much had he left?

9. A man had 8 rows of corn, with 10 hills in a row. The cows ate 50 hills. How many were left? Make a picture.

10. A man had 8 sheep in one pen and 6 in another. Five of them got away. How many were left?

11. $5+3-4+9+3-5-4+9=?$

12. $5\times 7+7-2=?$

13. Add, subtract, and multiply the numbers in each of these pairs:

4, 7 5, 8 10, 6 17, 2 5, 9 11, 4 6, 6 9, 4

14. What will 7 tables cost at 8 dollars each?

15. How many days in 8 school weeks?

16. There are 8 quarts in a peck. How many quarts in 7 pecks?

17. A boy gave 6 cents for an orange, 8 cents for pencils, and 10 cents for a ball. How many cents did he pay for all?

18. Add 9, 7, and 8, and then subtract 5.

DIVISION

28. Children watch. Teacher take 4 counters. Lay down two counters, and two more. Thus, || ||. Children do the same.

What did you do?

How many times did you lay down two counters?

How many counters did you take?

Take 4 crayons. Give me two crayons. Give me two more. How many times did you give me two crayons?

How many 2's in 4?

Take 6 sticks. Lay 2 of them. Lay 2 more. Lay 2 more. Thus, || || ||. How many times did you lay down 2 sticks?

Make 6 marks. Point them off in twos. Thus, ||, ||, ||.

How many groups are there?

How many times must you make 2 marks to have 6 marks?

How many 2's in 6?

So on till you have taught "there are ten 2's in 20."

Children write the table thus:

$2 \div 2 = 1$	$12 \div 2 = 6$
$4 \div 2 = 2$	$14 \div 2 = 7$
$6 \div 2 = 3$	$16 \div 2 = 8$
$8 \div 2 = 4$	$18 \div 2 = 9$
$10 \div 2 = 5$	$20 \div 2 = 10$

How many pears on the table in the picture?

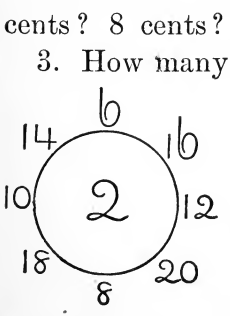
To how many children can the girl give the six pears, if she gives 2 pears to each child?

To how many sick people can she give the 6 roses, if she gives 3 roses to each?

1. A boy has 4 cents. How many pencils can he buy at 2 cents apiece?

2. A boy can earn 2 cents in an hour. In how many hours can he earn 10 cents? 14 cents? 18 cents? 6 cents? 8 cents? 16 cents? 20 cents?

3. How many 2's in 10? In 18? In 12? In 20?



BLACKBOARD DRILL

Divide each of the numbers around the circle by 2. Answer quickly.

29. Teacher and children each take 6 counters.

Lay 3 of them. Lay 3 more. Thus, III III.
Children tell what has been done.

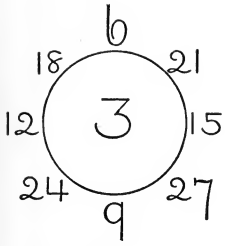
How many counters in each group?
How many groups?
How many groups of three-counters in six counters?
How many groups of 3-crayons in 6 crayons?
How many 3-cents in 6 cents?

How many apples are in the first row? How many in the second row?
Tell two ways of making 6.
How many 3's in 6?

Similarly with 9, 12, etc., counters.

Children write the table thus:

$3 \div 3 = 1$	$18 \div 3 = 6$
$6 \div 3 = 2$	$21 \div 3 = 7$
$9 \div 3 = 3$	$24 \div 3 = 8$
$12 \div 3 = 4$	$27 \div 3 = 9$
$15 \div 3 = 5$	$30 \div 3 = 10$



BLACKBOARD DRILL

Divide each of the numbers around the circle by 3.

1. How many pencils at 3 cents each will 12 cents buy?

2. How many suits can be made from 18 yards of cloth if it takes 3 yards for one suit?

3. How many yards in 30 feet?

4. Twenty-one boys stood in three equal lines for exercises. How many boys were in each line?

5. If a man ploughs 3 acres in one day, in how many days can he plough 24 acres?

In a similar manner, teach the entire table of division.

30. TABLE OF DIVISION

1÷1= 1	2÷2= 1	3÷3= 1	4÷4= 1	5÷ 5= 1
2÷1= 2	4÷2= 2	6÷3= 2	8÷4= 2	10÷ 5= 2
3÷1= 3	6÷2= 3	9÷3= 3	12÷4= 3	15÷ 5= 3
4÷1= 4	8÷2= 4	12÷3= 4	16÷4= 4	20÷ 5= 4
5÷1= 5	10÷2= 5	15÷3= 5	20÷4= 5	25÷ 5= 5
6÷1= 6	12÷2= 6	18÷3= 6	24÷4= 6	30÷ 5= 6
7÷1= 7	14÷2= 7	21÷3= 7	28÷4= 7	35÷ 5= 7
8÷1= 8	16÷2= 8	24÷3= 8	32÷4= 8	40÷ 5= 8
9÷1= 9	18÷2= 9	27÷3= 9	36÷4= 9	45÷ 5= 9
10÷1=10	20÷2=10	30÷3=10	40÷4=10	50÷ 5=10

TABLE OF DIVISION (*Continued*)

6÷6= 1	7÷7= 1	8÷8= 1	9÷9= 1	10÷10= 1
12÷6= 2	14÷7= 2	16÷8= 2	18÷9= 2	20÷10= 2
18÷6= 3	21÷7= 3	24÷8= 3	27÷9= 3	30÷10= 3
24÷6= 4	28÷7= 4	32÷8= 4	36÷9= 4	40÷10= 4
30÷6= 5	35÷7= 5	40÷8= 5	45÷9= 5	50÷10= 5
36÷6= 6	42÷7= 6	48÷8= 6	54÷9= 6	60÷10= 6
42÷6= 7	49÷7= 7	56÷8= 7	63÷9= 7	70÷10= 7
48÷6= 8	56÷7= 8	64÷8= 8	72÷9= 8	80÷10= 8
54÷6= 9	63÷7= 9	72÷8= 9	81÷9= 9	90÷10= 9
60÷6=10	70÷7=10	80÷8=10	90÷9=10	100÷10=10

1. $15 \div 3 = ?$	14. $40 \div 10 = ?$
2. $15 \div 5 = ?$	15. $40 = 10 \times ?$
3. $15 = 5 \times ?$	16. $40 = 4 \times ?$
4. $15 = 3 \times ?$	17. $27 = 9 \times ?$
5. $56 = 8 \times ?$	18. $27 = 3 \times ?$
6. $56 = 7 \times ?$	19. $27 \div 3 = ?$
7. $56 \div 7 = ?$	20. $27 \div 9 = ?$
8. $56 \div 8 = ?$	21. $72 \div 9 = ?$
9. $54 \div 9 = ?$	22. $72 \div 8 = ?$
10. $54 \div 6 = ?$	23. $72 = ? \times 8$
11. $54 = 9 \times ?$	24. $72 = 9 \times ?$
12. $54 = 6 \times ?$	25. $70 \times ? = 70$
13. $40 \div 4 = ?$	

DIVISION DRILL CHART

This chart contains all divisions in which the dividend does not exceed 100 and the divisor and quotient do not exceed 10.

It should be used daily till mastered. Vary the work, as suggested with the previous drill charts. Drill persistently.

10)100	6)30	9)90	1)1	6)18	9)9	1)4	6)60	6)54	7)28
7)42	1)5	6)24	9)81	3)9	6)12	5)15	10)20	4)24	8)40
2)18	9)27	6)36	10)90	2)14	10)10	4)20	9)36	2)2	6)48
7)35	3)30	8)56	2)16	9)18	3)6	9)72	3)3	10)60	5)10
1)6	10)30	5)40	6)6	5)20	8)48	7)70	7)21	8)64	2)4
6)42	7)28	3)12	9)63	1)2	10)40	3)15	9)45	3)18	10)80
7)49	8)16	1)10	3)27	7)63	2)6	8)24	4)8	7)14	4)4
1)3	4)16	10)50	2)12	4)12	9)54	2)10	5)45	3)21	8)32
7)56	8)8	5)35	5)25	3)24	8)72	1)8	10)70	1)7	5)5
4)40	4)36	5)30	1)9	8)80	2)8	5)50	2)20	4)32	7)7

31. 1. Take 24 counters. Divide them into 4 equal groups. How many are there in each group? How many 4's are there in 24? How many 6's are there in 24?

2. Take 15 counters. Divide them into 3 equal parts. How many are there in each part? How many 3's in 15? How many 5's in 15?

3. Two pints will fill a quart measure. How many

quart measures will 8 pints fill? 12 pints? 6 pints? 18 pints? 20 pints? 16 pints? 14 pints? 24 pints?

4. An orchard of 28 trees has 7 trees in each row. How many rows are there? Make a picture of the orchard.

5. Make a picture of an orchard having 30 trees with 10 trees in each row. How many rows are there?

6. A man has 72 peaches in 9 baskets, the same number in each basket. How many peaches in each basket?

7. If a man puts 4 plums in a row in a basket, how many rows would hold 24 plums? Make a picture, thus,

8. Make a picture of 48 birds with 8 in each flock. How many flocks?

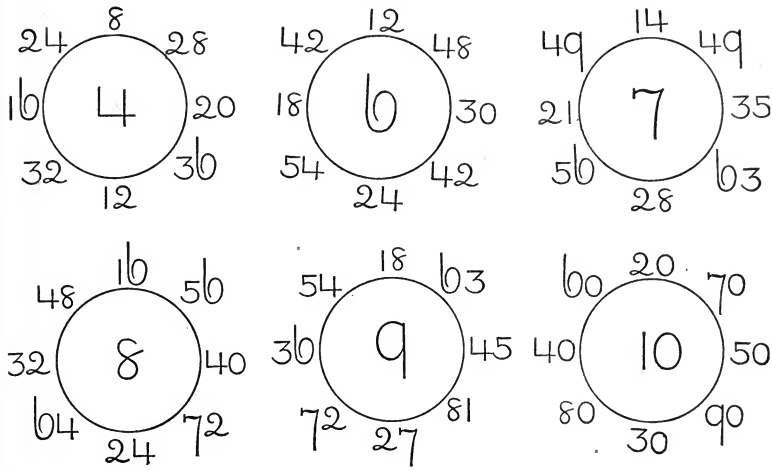
9. A man had 81 cents. He divided them among some boys, giving each boy 9 cents. How many boys were there?

10. A man's wages are 18 dollars per week. How much a day does he get?

11. Divide 63 sheep into 7 equal flocks. How many in each flock? Make a picture.

12. If you can buy 5 sheets of paper for a cent, how many cents will pay for 50 sheets?

BLACKBOARD DRILL



32. How many apples do you see here?

If these 8 apples are divided equally between two boys, how many apples will each boy receive?

How many apples are one half of 8 apples?

1. 10 dollars ÷ 2 are how many dollars?
2. $\frac{1}{2}$ of 10 dollars are how many dollars?
3. How do you find $\frac{1}{2}$ of 10 dollars?
4. 8 ÷ 2 are how many?
5. $\frac{1}{2}$ of 8 are how many?
6. How do you find $\frac{1}{2}$ of 8?
7. 16 quarts divided by 2 are how many quarts?
8. $\frac{1}{2}$ of 16 quarts are how many quarts?

9. How do you find $\frac{1}{2}$ of a number?
10. Find $\frac{1}{2}$ of 12.
11. Find $\frac{1}{2}$ of 6 pencils.
12. Find $\frac{1}{2}$ of 18 cents.
13. Find $\frac{1}{2}$ of 4.

14. If these 9 apples are divided equally among 3 boys, how many will each receive?
15. $\frac{1}{3}$ of 9 apples are how many apples?
16. 9 bushels $\div$ 3 are how many bushels?
17. $\frac{1}{3}$ of 9 bushels are how many bushels?
18. How do you find $\frac{1}{3}$ of a number?
19. Find $\frac{1}{3}$ of 21.
20. Find $\frac{1}{3}$ of 30.
21. Find $\frac{1}{3}$ of 12.
22. Find $\frac{1}{3}$ of 6.
23. Find $\frac{1}{3}$ of 24.
24. Find $\frac{1}{3}$ of 27.
25. Find $\frac{1}{3}$ of 15 days.
26. Find $\frac{1}{3}$ of 3.

33. 1. 8 quarts $\div$ 4 are how many quarts?
2. $\frac{1}{4}$ of 8 quarts are how many quarts?
3. 12 $\div$ 4 are how many?
4. $\frac{1}{4}$ of 12 are how many?
5. 20 cents $\div$ 4 are how many cents?
6. $\frac{1}{4}$ of 20 cents are how many cents?
7. $\frac{1}{4}$ of 24 are how many?
8. How do you find $\frac{1}{4}$ of a number?
9. Find $\frac{1}{4}$ of 16.
10. Find $\frac{1}{4}$ of 40 pounds.
11. Find $\frac{1}{4}$ of 32 apples.
12. Find $\frac{1}{4}$ of 28 pounds.
13. Find $\frac{1}{4}$ of 36 pounds.
14. Find $\frac{1}{4}$ of 4.
15. Divide 15 by 5.
16. $\frac{1}{5}$ of 15 are how many?

17. $30 \div 5$ are how many?
18. Find $\frac{1}{5}$ of 30 marbles.
19. How do you find $\frac{1}{5}$ of a number?
20. Find $\frac{1}{5}$ of 40 feet.
21. Find $\frac{1}{5}$ of 10 toes.
22. Find $\frac{1}{5}$ of 35 houses.
23. Find $\frac{1}{5}$ of 45.

34. 1. How can you find $\frac{1}{6}$ of a number? $\frac{1}{7}$ of a number? $\frac{1}{8}$? $\frac{1}{9}$? $\frac{1}{10}$?

2. Find $\frac{1}{8}$ of 40.
3. Find $\frac{1}{7}$ of 49.
4. Find $\frac{1}{9}$ of 36.
5. Find $\frac{1}{6}$ of 54.
6. Find $\frac{1}{10}$ of 100.
7. Find $\frac{1}{5}$ of 45.

8. Hugh has 20 rabbits and Ralph has $\frac{1}{2}$ as many. How many has Ralph?

9. Edna has 4 dolls and Susan has $\frac{1}{4}$ as many. How many dolls has Susan?

10. Mr. Smith walked 48 miles and Mr. Fox walked $\frac{1}{8}$ as far. How many miles did Mr. Fox walk?

11. William had 40 marbles and Frank had $\frac{1}{10}$ as many. How many had both?

12. There were 50 men in a factory. $\frac{1}{5}$ of them were black. How many were white?

13. Kate solved 42 problems; Ruth solved $\frac{1}{7}$ as many. How many did Ruth solve?

35. 1. How many twos in eight?

2. How many threes in ten, and how many over?

3. $\frac{1}{2}$ of 10 men are how many men?

4. Two mice have 8 feet. How many feet has one mouse?

5. If 4 oranges cost 8 cents, how many cents does one orange cost?

6. If 3 balls cost 9 cents, what is the cost of 1 ball?

7. How many nickels in a dime?

8. How many two-cent pieces in ten cents? In 8 cents?

9. $2\overline{)10}$ $4\overline{)8}$ $3\overline{)9}$ $5\overline{)10}$ $2\overline{)8}$ $3\overline{)6}$

10.	3×3	$8 - 5$	$4 + 4$	$10 \div 2$
11.	$8 - 6$	$11 - 4$	3×2	$9 \div 3$
12.	$5 + 3$	$6 + 5$	5×2	$\frac{1}{2}$ of 6
13.	$7 + 2$	$11 - 5$	$6 + 4$	$\frac{1}{4}$ of 8

14. If 5 pencils cost 10 cents, what does one pencil cost?

15. Put 8 sheep in 4 pens. How many sheep in each pen? (Picture.)

16. How many two-cent stamps can I buy for 8 cents?

17. Make problems for the numbers in Ex. 10. Ex. 11. Ex. 12.

18. Make a problem for $\frac{1}{2}$ of 6.

19. Make a problem for $\frac{1}{4}$ of 8.

36. 1. Eight boys and one boy are how many boys?

2. One ox and 5 oxen are how many oxen?

3. If 1 sheep costs \$3, what will 3 sheep cost?

NOTE. — Teach the use of the dollar mark.

4. How many knives can I buy for $10, if one knife costs $2?

5. If 4 geese cost $8, what does 1 goose cost?

6. Add

3	5	6	5	7	3	5
2	3	4	5	3	0	4

7. Subtract

8	9	10	10	9	6	8
−3	−5	−6	−10	−6	−3	−2

8. 4×2 8×1 2×4 10×0 2×3

9. 6×1 3×3 5×2 10×1 $8 \div 4$

10. $5)\underline{10}$ $4)\underline{8}$ $3)\underline{6}$ $2)\underline{10}$ $4)\underline{4}$ $2)\underline{8}$

11. Mary has two dolls, and Annie has nine. How many have both?

12. There are 5 chairs in one row and 6 in another. How many chairs in all?

13. John had 10 balls and lost three. How many had he left?

14. How many cents are 2 cents, 3 cents, and 5 cents?

15. How many boys are 2 boys, 4 boys, and 5 boys?

37. 1. A window had 10 panes of glass and a boy broke 3 panes. How many were left?

2. If a pail holds 6 quarts, and there are 5 quarts of milk in it, how many quarts more will it hold?

3. Four teams of goats are how many goats?

4. How many pair will 10 shoes make?

5. There were 10 white mice in a box and 7 ran out. How many were left in the box?

6. What 3 pieces of money make 5 cents?

7. How many three-cent cakes can you buy for 10 cents, and how much will you have left?

8. If a six-quart pail is half full, how many quarts are in the pail?

9. How much less than a dime is 3 cents?

10. 3 books cost $6. What will one book cost? What will 5 books cost?

11. I bought an orange for 3 cents and candy for 5 cents. How much money did I spend?

12. $\frac{1}{2}$ of 6 is what? $\frac{1}{4}$ of 8 is what?

13. $\frac{1}{5}$ of 10 is what? $\frac{1}{2}$ of 10 is what?

14. $3 \times ? = 9$. $9 \div 3 = ?$ $10 - 4 = ?$ $6 + 3 = ?$

15. $? + 4 = 8$. $8 \div 4 = ?$ $9 - ? = 4$. $10 - ? = 8$.

16. How do you find $\frac{1}{3}$ of a number? $\frac{1}{4}$? $\frac{1}{5}$?

38. 1. Add

(2)	(3)	(4)	(5)	(6)	(7)
6	2	5	1	4	3
3	3	2	7	5	3
1	4	3	2	1	3

8. $5 + 3 - 1 = ?$ $3 + 3 - 2 = ?$ $2 + 2 + 2 = ?$

9. $6 - 4 + 2 = ?$ $8 - 4 - 2 = ?$ $3 + 3 + 3 = ?$

10. $\frac{1}{2}$ of 10, $\frac{1}{6}$ of 6, 3×3, 5×2.

11. $\frac{1}{4}$ of 8, $\frac{1}{3}$ of 9, 4×2, $8 \div 4$.

12. Ned lives 6 blocks east of the school, and Fred 4 blocks east. How far from Ned's to Fred's? (Picture.)

13. John lives 4 blocks west of the school. How far from John's to Ned's? From Ned's to John's? (Picture.)

14. One week and 5 days are how many days?

15. Nellie had 5 two-cent pieces. She bought a book for 4 cents, and a pencil for 3 cents. How much money did she spend? How much was left?

16. May had ten cents and spent one-fifth of it for a pencil. How much did the pencil cost, and how much has she left?

17. From three 3's take two 4's.

18. One book costs $4. How much will 2 books cost?

39. 1. A farmer had 11 turkeys and sold 6 of them. How many had he left?

2. If there are two nests, with 5 eggs in each nest, how many eggs in both nests? (Picture.)

3. If George had 10 cents and gave $\frac{1}{5}$ to his sister, how much will she have? How much will he have left?

4. Rose had 3 apples; her brother had twice as many. How many had her brother? How many had both?

5. A hen had 11 chickens; 2 were black, 4 were brown, and the rest were white. How many were white?

6. A lady bought 11 yards of cloth, and used 6 yards. How many has she left?

7. May is 11 years old. Maud is 4 years younger. How old is Maud?

8. $3+4+2$ $8-3-2$ $5\times 2+1$

9. $2+7+2$ $7+4-6$ $3\times 3+2$

10. $2+2+7$ $6+5-8$ $4+4+3$

11. Add

(12)	(13)	(14)	(15)	(16)	(17)
3	1	2	3	2	3	4
4	3	2	3	4	1	2
2	5	2	3	3	3	3
2	2	5	2	1	1	2

40. 1. One tooth and 11 teeth are how many teeth?

2. A dime and a two-cent piece make how much?

3. Six two-cent pieces are how much money?

4. How many cents in 4 three-cent pieces?

5. How many 2-cent stamps can you buy for 12 cents?

6. In 12 pencils how many dozen?

7. Find the cost of 4 oranges at 3 cents apiece.

8. If one hat costs $ 6, what will 10 hats cost?

9. Mary has 9 cents, Lillie has 6 times as much. How much has Lillie?

10. A pint of milk costs 2 cents. How many pints will 12 cents buy?

11. 4 triangles have how many angles? (Picture.)

12. Four men and 18 men are how many men?

13. Twelve eggs less 7 eggs are how many eggs?

14. Three times four balls are how many balls?

15. Three squares have how many sides? (Picture.)

16. Twelve feet less one foot are how many feet?

17. If there are 12 plants in four rows, how many plants in each row? (Picture.)

18. Fred had 12 marbles. He gave 3 to Henry and 2 to Tom. How many had Fred left?

19. Six boys can wear how many shoes?

41. 1. 7 boys can wear how many shoes?

2. How many 2-cent pieces in 14 cents?

3. If one dress costs $ 7, what will 2 dresses cost?

4. In 14 days how many weeks?

5. From 14 cents take 9 cents. How many cents are left?

6. If you can buy 1 peach for 2 cents, what will 7 peaches cost?

7. Mary has 4 three-cent pieces and a 2-cent piece. How much money has she?

8. There were 14 children in a class; 9 were girls. How many were boys?

9. If it takes 7 yards of cloth for one dress, how many dresses can be made from 14 yards?

10. Mrs. Smith had $ 14 and spent $ 8 for a dress. How many dollars were left?

11.	1×15	$\frac{1}{5}$ of 15	$15 - 4$
12.	$15 \div 15$	$\frac{1}{3}$ of 15	$9 + 6$
13.	15×1	$13 + 2$	$15 - 9$
14.	3×5	$3 + 13$	$15 - 15$

15. Make problems for:

3×5 $\frac{1}{3}$ of 15 $13 + 2$

42. 1. There are 7 red apples and 10 green ones in a basket. How many apples are in the basket?

2. Frank had 9 balls and bought 8 more. How many had he then?

3. Three five-cent pieces and a two-cent piece are how many cents?

4. If Henry finds 9 eggs in one nest and 8 eggs in another, how many eggs does he find in all? (Picture.)

5. A farmer has 6 cows in one yard and 11 in another. How many cows in both yards? (Picture.)

6. In a flock of 17 sheep 6 sheep are black. How many are white?

7. Frank paid 17 cents for a top and sold it for 8 cents. How much did he lose?

8. A lady bought a book for $17 and paid $9. How much did she still owe?

9.	$16 \div 1$	$15 + 1$	$\frac{1}{2}$ of 16
10.	1×16	$16 - 15$	$\frac{1}{4}$ of 16
11.	$16 \div 8$	$16 - 1$	$\frac{1}{3}$ of 15
12.	16×1	$14 + 2$	$\frac{1}{5}$ of 15
13.	4×4	$16 - 14$	$11 + 5$
14.	$16 \div 4$	$2 + 14$	$10 + 6$
15.	2×8	$13 + 3$	$16 - 9$
16.	$16 \div 2$	$13 - 13$	$16 - 0$
17.	8×2	$12 + 4$	$4 + 12$

18. Count to 16 by 2's; by 4's.

19. Make problems for $16 \div 4$, $\frac{1}{4}$ of 16, $16 \div 8$.

43. 1. May is 10 years old; John is 9 years older. How old is John?

2. May had 6 cents, Lucy had 8 cents, Ella had 5 cents. How much had they all?

3. A pole was 19 feet long. I cut off 6 ft. at one time and 10 ft. at another time. How many feet were cut off and how many were left? (Picture.)

4. From nineteen take nine.

5. I spent $4 for books, $10 for a case, and had $5 left. How much had I at first?

6. 19 children were at school and 7 of them went home. How many are left?

7. Jane had 9 roses; her sister had 4 more. How many must they pick to have 19 roses in all?

8.	$5+?=14$	$?\times 6=18$	$\frac{1}{3}$ of $18=?$
9.	$16-8=\ ?$	$?-9=\ 6$	$\frac{1}{4}$ of $16=?$
10.	$14-5=\ ?$	$12-?=\ 5$	$\frac{1}{2}$ of $18=?$
11.	$7+?=13$	$17-?=\ 8$	6 is $\frac{1}{2}$ of ?
12.	$16+2=\ ?$	$?\div 2=\ 8$	4 is $\frac{1}{4}$ of ?
13.	5×4	$20-3$	$\frac{1}{2}$ of 20
14.	$20\div 5$	$16+4$	$\frac{1}{5}$ of 15

15. Add

16	18	17	15	8	4	5	2
4	2	3	5	12	14	12	13

16. $5)\underline{20}$ $4)\underline{20}$ $2)\underline{16}$ $9)\underline{18}$ $3)\underline{15}$ $4)\underline{12}$

44. 1. Take 3×4 from 2×9.

2. If my newspaper comes twice a week, how many papers will I get in 8 weeks?

3. A boy had 18 apples. He gave away $\frac{1}{3}$ of them and lost 3. How many had he left?

4. 7 is $\frac{1}{2}$ of ? 12 is twice what?

5. 4 is $\frac{1}{4}$ of ? 18 is three times what?

6. A hen had a dozen chickens. Six were black and the rest were white. How many were white?

7. How many thumbs have four pair of mittens?

8. Count to 18 by 3's.

9. A ten-cent piece and a two-cent piece will buy how many marbles at 4 cents each?

10. 13 birds were on a tree and 7 flew away. How many remained?

11. Nellie had 19 apples and gave away 4 of them and ate 1. How many had she left?

12. How many twos in 20?

13. $2 \times 8 \div ? = 4$

14. $18 = 3 \times 5 + ?$

15. $4 \times 2 \times 2 = ?$

16. $\frac{1}{2}$ of $20 = 5 \times ?$

17. $7 \times 2 + 3 = ?$

45. DRILL EXERCISE

1. $\frac{1}{2}$ of $20 = ?$
2. $\frac{1}{5}$ of $? = 3$
3. $5 \times 3 + 2 = ?$
4. $\frac{1}{2}$ of $14 = ?$
5. $\frac{1}{2}$ of $? = 8$
6. $20 - \frac{1}{2}$ of $14 = ?$

$3 \times 4 = ?$

$7 + 11 = ?$

$\frac{1}{3}$ of $18 = ?$

$\frac{1}{2}$ of $18 = ?$

$19 - 7 = ?$

$20 \div 10 = ?$

7. Add

8	(8) 7	(9) 6	(10) 2	(11) 7	(12) 9
3	4	5	4	2	9
5	5	7	6	5	1
4	4	1	8	6	2

13. From 20 take 11, 4, 6, 12, 9, 8.

14. $\frac{1}{4}$ of 16 = ? $\frac{1}{3}$ of 18 = ? $\frac{1}{6}$ of 18 = ?

15. $\frac{1}{4}$ of 12 = ? $\frac{1}{3}$ of 12 = ? $\frac{1}{5}$ of 15 = ?

16. To 7 add 4, 12, 8, 9, 3, 5.

17. 2)20 9)18 2)16 3)15 4)16

18. Divide 50, 60, 70, 80, 90 — each by 10.

19. 5 in 15, 20, 10, 5, 0.

20.	$20 \div 1$	$19 + 1$	$16 + 4$
21.	1×20	$20 - 1$	$20 - 16$
22.	$20 \div 20$	$1 + 19$	$4 + 16$
23.	20×1	$20 - 19$	$20 - 4$
24.	2×10	$18 + 2$	$15 + 5$
25.	$20 \div 2$	$20 - 18$	$20 - 15$
26.	$29 - 10$	$20 - 2$	$5 + 15$

46. 1. Find the cost of 8 pair of shoes at $3 a pair.

2. If a man earns $12 a week, how much can he earn in 2 weeks?

3. How many eggs in 2 dozen?

4. How many two-quart pitchers will 24 quarts of water fill?

5. How many threes in 2 dozen?

6. What number is $\frac{1}{2}$ of 24?

7. Find the cost of 6 books at \$4 each.

8. A lady bought a dozen eggs. $\frac{1}{3}$ of them were bad. How many were good?

9. My square table is 6 feet on a side. How many feet of fringe will it take to go around it? (Picture.)

10. Bought 6 lemons for 24 cents. What did one lemon cost?

11. Two weeks and 10 days are how many days?

12. How many feet have 3 cats and 3 rats?

13. What four numbers added together make 21?

14. $\frac{1}{6}$ of 12 ct. = ? $\frac{1}{4}$ of \$12 = ? $\frac{1}{2}$ of 18 = ?

15. $7\overline{)21}$ $6\overline{)18}$ $3\overline{)15}$ $4\overline{)16}$ $3\overline{)21}$ $8\overline{)16}$

16.

8	7	6	2	4	8	7	9	8
×2	×3	×3	×9	×4	+5	×3	+7	+6

17.

17	18	14	17	15	7	9
−6	−9	−2	−6	−2	×3	+4

18. $\frac{1}{2}$ of 20 = ? $\frac{1}{3}$ of 15 = ? $\frac{1}{5}$ of 15 = ?

19. $\frac{1}{6}$ of 18 = ? $\frac{1}{4}$ of 20 = ? $\frac{1}{3}$ of 21 = ?

20. What three equal numbers in 21?

21. What seven equal numbers in 21?

47. 1. $5 \times 5 = ?$

2. $25 \div 5 = ?$

3. $\frac{1}{5}$ of 25 = ?

Add 4, 5, 3, 5, 5.

$6 + 4 + 5 + 6 + 1 = ?$

4. $\frac{1}{5}$ of $15 = ?$ $\frac{1}{6}$ of $24 = ?$

5. $18 + ? = 25$ $3 \times 8 = ?$ $5 \times 3 = ?$

6. $25 \div 5 = ?$ $24 \div 8 = ?$

7. Harry bought a knife for 15 cents and sold it for 25 cents. What did he gain?

8. There are 25 apples in a basket. I take $\frac{1}{5}$ of them. How many do I take? How many are left?

9. How many 3-cent pieces in 27 cents?

10. How many nickels in a quarter?

11. How many cents in nine nickels?

12. Give 3 cents to each of nine children. How much do all get?

13. John bought 3 dozen buttons at 9 cents a dozen. How much did they cost?

14. If 9 books cost $27, what does one book cost?

15. How many 9's in 27?

16. There are 3 feet in a yard. How many yards in 27 feet? (Picture.)

48. 1. A table has four sides; each side is 7 feet long. How many feet around the table? (Picture.)

2. 7 oranges cost 28 cents. What is the cost of one orange?

3. What will 4 books cost at 7 cents each?

4. 7 pair of oxen have how many horns?

5. If there are 4 trees in a row, how many trees in 7 rows? (Picture.)

6. What three equal numbers make twenty-four?

7. What month has 28 days? When does it have 29 days?

8. If I make $4 in selling a stove, how many stoves must I sell to make $28?

9. How many school days in 6 weeks?

10. Into what three equal numbers can 30 be divided?

11. A milkman had five six-quart cans of milk. How much milk had he?

12. How many 5's in 3×10?

13. $10 is $\frac{1}{3}$ of how much money?

14. How many two-cent stamps can I buy for 30 cents?

15. What piece of money is $\frac{1}{6}$ of 30 cents?

16. John has 30 cents, and Mary has $\frac{1}{6}$ as much. How much has Mary?

17. If 8 pencils cost 32 cents, what does 1 pencil cost? What will 7 pencils cost?

18. If 32 cents are divided equally among 8 children, how many cents will each receive?

19. What will 8 pounds of rice cost at 4 cents a pound?

49. 1. How many 8's in 35, and how many over?

2. In 36 feet how many yards? (Picture.)

3. How many boys will it take for 4 baseball nines? (Picture ball ground.)

4. How many months in 3 years?

5. What cost 9 lemons at 36 cents a dozen?

6. A man earns $9 a week. How much does he earn in 4 weeks?

7. How many 4-dollar chairs can be bought for $36?

8. How many eggs in 3 dozen?

9. Three boys earned 36 cents. How much was that for each one?

10. What two equal pieces of money make $\frac{1}{6}$ of 36 cents?

11.	$5 \times 7 = ?$	$\frac{1}{5}$ of 35	$7 \times ? = 35$
12.	$35 \div 5 = ?$	$\frac{1}{7}$ of 28	$6 \times ? = 30$
13.	$35 \div 7 = ?$	$\frac{1}{4}$ of 24	$8 + ? = 30$

14. $4+6+9+6+3=?$ 17. $6+8+9+7+5=?$

15. $7+4+3+7+2=?$ 18. $2+7+6+4+6=?$

16. $9+3+6+2+9=?$ 19. $7+7+7+7+7=?$

20. Make seven rows of squares with five squares in a row. How many squares? How many angles have $\frac{1}{5}$ of these squares?

21.	$\frac{1}{2}$ of 16	$\frac{1}{3}$ of 39	$16 \div 8$	
22.	$\frac{1}{4}$ of 32	$\frac{1}{4}$ of 40	$14 \div 2$	
23.	$\frac{1}{9}$ of 27	$\frac{1}{6}$ of 36	$28 - 14$	$38 - 18$
24.	$\frac{1}{8}$ of 32	$\frac{1}{10}$ of 40	$36 - 18$	
25.	$\frac{1}{7}$ of 21	$\frac{1}{5}$ of 25	$27 \div 9$	$36 - 20$

50. 1. In 16 dimes how many cents, and how many over?

2. A father gave to one of his sons 30 cents, to another 14 cents, and to his daughter 2 cents. How much did he give them in all?

3. I bought a stove for $\$40$ and a table for $\frac{1}{4}$ as much. How much did both cost?

4. Count from 3 to 45 and back by sixes.

5. Find the sum of 7 times 4 and 4 times 2.

6. Add

3	(7) 9	(8) 9	(9) 6	(10) 4	(11) 6
5	4	8	11	6	5
10	6	7	9	9	11
11	3	6	3	6	5
5	2	5	8	4	9
7	1	4	7	5	9

12. Multiply

5	7	8	6	9	4	7	5	6
9	6	4	7	3	8	7	10	6

13. 36 is how many times 6?

14. 45 is 9 times what number?

15. $45-9+3=?$

16. $45-15+10=?$ $49-7=?$

17. $\frac{1}{5}$ of 45 5×8

18. $\frac{1}{9}$ of 45 7×6

19. $\frac{1}{4}$ of 44

20. $\frac{1}{7}$ of 42

21. $\frac{1}{8}$ of 48

51. 1. How many cents in 6 dimes?

2. How many school days in 8 weeks?

3. How many working days in 5 weeks?

4. In 48 eggs how many dozen?

5. A lady bought a shawl for $\$30$ and a dress for $\frac{1}{3}$ as much. What did both cost?

6. If I buy some cloth for $44 and sell it at a loss of $13, what do I get for it?

7. If a pound of candy costs 12 cents, what will 4 pounds cost?

8. How many nickels in $.45?

NOTE. — Teach how to indicate cents by means of the decimal point and dollar sign.

9. If 4 quarts of vinegar can be bought for 28 cents, how many quarts can be bought for 49 cents?

10. Mr. Smith gave $50 to his five boys. How much did each get?

11. $\frac{1}{4}$ of $.40 $3 \times \$.09$

12. $\frac{1}{6}$ of $.42 $4 \times \$.11$

13. $\frac{1}{7}$ of $.49 $8 \times \$.06$

14. $\frac{1}{8}$ of $.48 $4 \times \$.12$

15. $\frac{1}{5}$ of $.45

16. $\frac{1}{6}$ of $.48 $5 \times \$.11$ $4 \times 8 + 6$

17. $45 \div ? = 9$ $? \times 6 = 48$

18. $49 \div 7 = ?$ $? + 15 = 45$

19. From 9×5 take 7×6.

20. How many days does a man work in nine weeks?

21. A load of potatoes had 54 bushels. I bought $\frac{1}{9}$ of the load. How many bushels have I?

22. How many hats can a man buy for $54 if 4 hats cost $24?

52. 1. If 8 bars of soap cost $.56, what is the cost of one bar?

2. What is the cost of 9 lb. of sugar at 6 ct. a pound?

3. Willie has 20 marbles, James 20, and John 10. How many have all?

4. May had $.59. She spent 10 cents for paper. How much had she left?

5. Into how many lots of 7 acres each can a 56-acre lot be divided?

6. How many days in 6 weeks?

7. How many eggs in 4 dozen?

8. If I owe $.58 and pay all but 10 cents, how much have I paid?

9. Max bought a knife for 41 cents and sold it so as to gain 10 cents. What did he sell it for?

10.

10 + 1	10 + 2	10 + 3	10 + 4
20 + 1	20 + 2	20 + 3	20 + 4
30 + 1	30 + 2	30 + 3	30 + 4
40 + 1	40 + 2	40 + 3	40 + 4
50 + 1	50 + 2	50 + 3	50 + 4

11.

10 − 2	10 − 3	10 − 4	10 − 5
20 − 2	20 − 3	20 − 4	20 − 5
30 − 2	30 − 3	30 − 4	30 − 5
40 − 2	40 − 3	40 − 4	40 − 5
50 − 2	50 − 3	50 − 4	50 − 5

12. Count to 55 by fives.

53. 1. In a garden are 48 plants in 6 equal rows. How many plants in each row?

2. How many weeks must a man work to earn $48, if he earns $12 a week?

3. A boy paid $.40 for a knife and $.15 for a slate. What was the cost of both?

4. From a piece of calico containing 58 yards I cut 18 yards. How many yards were left?

5. If I earn $.09 a day for a week, how much have I earned?

6. At 8 cents each, what will 7 cocoanuts cost?

7. John lives 10 blocks east of the school, and William 15 blocks west. How far does John live from William? (Illustrate.)

8. If a ship sails 63 miles in 9 hours, how far will she sail in 1 hour?

9.	$11 + 4$	$11 - 4$	$12 - 5$	$12 + 5$
10.	$21 + 4$	$21 - 4$	$22 - 5$	$22 + 5$
11.	$31 + 4$	$31 - 4$	$32 - 5$	$32 + 5$
12.	$41 + 4$	$41 - 4$	$42 - 5$	$42 + 5$
13.	$51 + 4$	$51 - 4$	$52 - 5$	$52 + 5$

14. Count to 60 by 5's; by 3's; by 2's.

15.	9×5	8×4	7×5	9×7
16.	9×4	8×6	6×7	7×7
17.	9×6	8×5	7×8	6×6
18.	9×3	8×8	7×4	11×4

54. 1. How many dimes in 60 cents?

2. Bought 9 lamps at 4 dollars apiece, and sold them at 7 dollars apiece. How much did I gain?

3. What cost 2 oranges at 60 cents a dozen?

4. In one field there were 20 cows; in another twice as many. How many in the second field? How many in both fields?

5. If 4 apples cost 12 cents, how much will 9 apples cost?

6. What 7 equal numbers make 56?

7. If 8 bars of soap cost $.56, what is the cost of one bar?

8. What is the cost of 9 pounds of sugar at 6 cents a pound?

9. Willie has 20 marbles, James 20, and John 10. How many have all?

10.	$13 + 5$	$13 - 6$	$14 + 7$	$14 - 7$
11.	$23 + 5$	$23 - 6$	$24 + 7$	$24 - 7$
12.	$33 + 5$	$33 - 6$	$34 + 7$	$34 - 7$
13.	$43 + 5$	$43 - 6$	$44 + 7$	$44 - 7$
14.	$53 + 5$	$53 - 6$	$54 + 7$	$54 - 7$

15. Add 6, 9, 8, 7, 3, 4, 2, 15, 4.

16.

Add	(17)	(18)	(19)	(20)
8	6	12	10	4
12	8	2	5	6
10	4	8	4	10
8	9	11	6	5
9	12	9	1	5

55. 1. How many spools of thread at 4 cents a spool can you buy for 36 cents?

2. In an orchard are 8 rows of trees with 9 trees in each row. How many trees in the orchard?

3. I have five 10-pound jars of butter, and sell it all at a profit of 6 cents a pound. What do I make?

4. How many pounds of steak at 8 cents a pound can I buy for 32 cents?

5. If 60 cents be paid for thread at 4 cents a spool, how many spools are bought?

6. Count from 6 to 61 and back by fives.

7. Count from 8 to 62 and back by 6's.

8. How many suits of clothes can be made from 63 yards of cloth, if one suit takes 7 yards?

9. A wagon cost \$64, a harness $\frac{1}{8}$ as much. What did both cost?

10. At \$12 a week, how much will a man earn in 5 weeks?

11. If a pair of boots cost \$6, what will be the cost of 5 pair? 7 pair? 9 pair?

12. If a boy can ride a bicycle 6 miles in 6 hours, how far can he go in an hour?

13. At 7 cents a pound, how many pounds of sugar can I buy for 63 cents?

14. Count from 5 to 69 and back by fours.

15. Count from 28 to 72 and back by 4's.

16. 9 times 4; 6; 3; 2; 9; 8; 7; 10; 11; 12.

17. Count by nines to 108.

18.	$11+7$	$11-7$	$12+7$	$12-7$
19.	$21+7$	$21-7$	$22+7$	$22-7$
20.	$31+7$	$31-7$	$32+7$	$32-7$
21.	$41+7$	$41-7$	$42+7$	$42-7$

56. 1: How many 5-cent pieces in 50 cents?

2. How many eggs in 6 dozen?

3. What must I pay for 7 pencils at 4 cents each?

4. If 3 girls can do a piece of work in 6 days, in how many days can one girl do it?

5. If 5 cents is $\frac{1}{5}$ of my money, how much money have I?

6. How many months in 6 years?

7. How many boots in 1 dozen pair?

8. How many are 6 dozen oranges less 8 oranges?

9. What four pieces of money make 75 cents?

10. In 72 books how many dozen?

11. If it takes 9 yards of calico for one dress, what will it take for 8 dresses?

12.	9×8	12×6	$63\div 7$	$70-40$
13.	4×5	$72\div 9$	$15+15$	$75-25$
14.	3×12	$64\div 8$	$12+20$	$36-20$
15.	8×8	7×9	$18+10$	$50-25$

16. Count from 7 to 73 and back by sixes.

17. Count from 8 to 78 and back by sevens.

18.	$11+9$	$11-9$	$13+8$	$13-8$
19.	$21+9$	$21-9$	$23+8$	$23-8$
20.	$31+9$	$31-9$	$33+8$	$33-8$
21.	$41+9$	$41-9$	$43+8$	$43-8$
22.	$51+9$	$51-9$	$53+8$	$53-8$
23.	$61+9$	$61-9$	$63+8$	$63-8$

57. 1. How many dimes in 70 cents?

2. Twenty-eight horseshoes will shoe how many horses?

3. In our class there are seven children in each row. How many children in 8 rows?

4. Will had 35 papers, and Harry had twice as many. How many had Harry?

5. At 4 dollars a day, how much does a man earn in 2 weeks?

6. If I have $8 in quarters, how many quarters have I?

7. How many legs have a dozen flies?

8. At 7 cents each, how many tablets can I buy for 84 cents?

9. John had eighty-four apples. How many dozen did he have?

10. If 1 dozen oranges cost 48 cents, what does 1 orange cost?

11. $12 + 8 + 6 + 9 + 4 + 5 + 3 + 2 = ?$

12. $8 + 9 + 10 + 6 + 5 + 6 + 4 + 3 = ?$

13. $3 \times 3 \times 3 \times 3 = ?$ $2 \times 2 \times 2 \times 2 \times 2 \times 2 = ?$

14.	$2 + 9$	$52 + 9$	$52 - 9$	$6 + 5$
15.	$12 + 9$	$62 + 9$	$42 - 9$	$16 + 5$
16.	$22 + 9$	$72 + 9$	$32 - 9$	$26 + 5$
17.	$32 + 9$	$72 - 9$	$22 - 9$	$36 + 5$
18.	$42 + 9$	$62 - 9$	$12 - 9$	$46 + 5$

58. 1. How many school weeks in 40 days?

2. How much sugar at \$.08 a pound can I buy for \$.72?

3. Emma is 9 years old; her father is five times as old. What is her father's age?

4. If you earn 12 cents in a week, how much do you earn in a month?

5. How many quarts of oil do we use in 12 weeks if we use 1 quart each day?

6. There are 7 days in a week. How many days in 3 weeks and 5 days?

7. Ned has $\frac{1}{3}$ of 27 cents. George has $\frac{1}{4}$ of 28 cents. How many cents have both?

8. Eight chairs cost \$32. How much apiece do they cost?

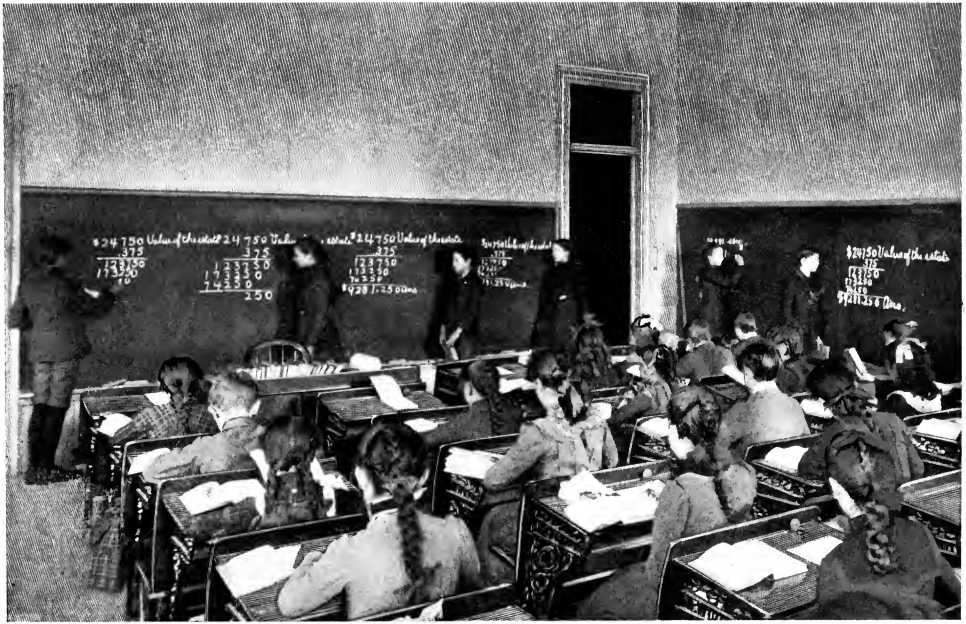
9. There are 12 things in a dozen. 4×9 are how many dozen?

10. Seven apples cut into quarters make how many quarters?

11. A lady had 32 apples. She gave 5 boys 6 apples apiece. How many had she left?

12. $72 \div 6 + ? = 20$ $100 \div 10 - ? = 1$

13.	7×9	$100 - 4$	$19 + 8$	$19 - 8$
14.	100×5	$96 \div 12$	$29 + 8$	$29 - 8$
15.	18×4	$72 \div 12$	$39 + 8$	$39 - 8$
16.	16×5	$88 \div 11$	$49 + 8$	$49 - 8$
17.	14×5	$72 \div 9$	$59 + 8$	$59 - 8$



PUPILS AT THE BLACKBOARD.

To face page 75.

PART SECOND

NOTATION AND NUMERATION

1. That which tells *how many* is **Number.** As, 11, 12 books, 15 cents.

2. One is a **Unit.** As, 1, 1 dollar, 1 horse.

3. Every number is made up of units. Three contains 3 units. Twenty contains 20 units. Fifty trees contains 50 units.

4. Writing numbers is **Notation.** As, 7, VII, seven.

5. Writing numbers in figures is **Arabic Notation.** As, 13.

6. In Arabic notation, ten figures are used in writing numbers. They are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. They are called naught, one, two, three, four, five, six, seven, eight, nine. The first figure is also sometimes called a cipher or zero.

One is written	1
Ten is written	10
One hundred is written	100
One thousand is written	1000
Ten thousand is written	10000
One hundred thousand is written . . .	100000
One million is written	1000000
Ten million is written	10000000
One hundred million is written	100000000

7. Read the above numbers. What would the numbers be if 5 were used where 1 is?

If 7 were used where 1 is? If 3 were used where 1 is?

How many ones are there in 10?
How many tens are there in 100?
How many hundreds are there in 1000?
How many thousands are there in 10,000?
How many ten-thousands are there in 100,000?
How many hundred-thousands are there in 1,000,000?
How many millions are there in 10,000,000?
How many ten-millions are there in 100,000,000?

What is the value of 0?

Since 0 has no value, what is it that has value in these numbers?

How does the value of the figure 1 in the second number compare with the value of the figure 1 in the first number?

The 1 in the third number with the 1 in the second number?

The 1 in the fourth number with the 1 in the third number?

The 1 in any of these numbers with the 1 in the number before it?

The *place* of the 1 in any of these numbers compares how with the place of the 1 in the number before it? Answer all these questions about the numbers, using 2, 3, 5, or 7 in place of 1.

What must you do with the 1, 2, 3, 5, or 7 in any of these numbers to get the number after it? That would affect the value of the figure how?

What would you have to do with the 1, 2, 3, 5, or 7 in any of these numbers to get the number before it? How would that affect the value of the figure?

On what, then, does the value of a figure depend?

For what are the ciphers used?

8. The answers to the foregoing questions show the following:

Principles of Arabic Notation

1. Each removal of a figure one place to the left increases its value tenfold.

2. Each removal of a figure one place to the right decreases its value tenfold.

3. The value of a figure depends upon its place in the number.

4. Ciphers are used to give the other figures their proper places.

100,000,000 = ONE HUNDRED MILLION
. 10,000,000 = TEN MILLION
. . 1,000,000 = ONE MILLION
} = 111 MILLION

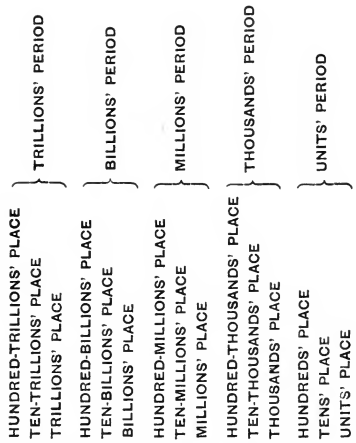
. . . 100,000 = ONE HUNDRED THOUSAND
. . . . 10,000 = TEN THOUSAND
. . . . . 1,000 = ONE THOUSAND
} = 111 THOUSAND

. . . . . 100 = ONE HUNDRED
. . . . . 10 = TEN
. . . . . 1 = ONE
} = 111 (UNITS)

111,111,111 = 111 MILLION, 111 THOUSAND, 111

ONE HUNDRED MILLION
TEN MILLION
MILLION
HUNDRED THOUSAND
TEN THOUSAND
THOUSAND
HUNDRED
TEN
UNIT

MILLIONS' PERIOD
THOUSANDS' PERIOD
UNITS' PERIOD



695,432,741,897,654 = six hundred ninety-five trillion, four hundred thirty-two billion, seven hundred forty-one million, eight hundred ninety-seven thousand, six hundred fifty-four.

320,105 = three hundred twenty thousand, one hundred five.

907,035,700 = nine hundred seven million, thirty-five thousand, seven hundred.

5,001,006 = five million, one thousand, six.

70,000,025 = seventy million, twenty-five.

13,000,000,500,000 = thirteen trillion, five hundred thousand.

11. **1.** Each figure occupies a place. How many places are there in a period?

2. Beginning at the right, name in order five periods.

3. How many figures are there in five periods?

4. How many places are there in five periods?

5. Name fifteen places in order, beginning at the right.

6. In reading numbers, when you come to a period composed of three ciphers, what do you read for that period?

7. Why is it necessary to use three ciphers in writing the number?

8. How does the name of each *period* compare with the name of its right-hand *place?*

9. Write a number containing one period.

10. Write a number containing three periods.

11. Write a number containing 12 places. How many periods are there in such a number?

NUMERATION

12. Naming the places of figures and reading numbers is **Numeration.** Thus, to numerate 43,008,160, you should say, "Units, tens, hundreds, thousands, ten-thousands, hundred-thousands, millions, ten-millions — forty-three million, eight thousand, one hundred sixty."

13. Numerate the numbers below:

1.	385	6.	35,000,730
2.	1,421	7.	8,460,000
3.	25,678	8.	423,000,501
4.	315,129	9.	8,003,040,631
5.	6,785,342	10.	8,900,760

14. Write in figures:

1. Two hundred thousand, sixteen.

2. Eleven thousand, two.

3. Four million, six hundred eight thousand, three hundred seventy-five.

4. Twenty-five thousand, three hundred eighty-seven.

5. Nineteen thousand, seventeen.

6. Twenty-seven million, six hundred fifty-two.

7. Eighty million, six hundred nine thousand, four hundred twenty-eight.

8. Four hundred thirty-six thousand, forty-one.

9. Six hundred twenty million, seventeen thousand, four hundred seventy-seven.

10. One hundred fifty-seven million, six hundred eight thousand, four hundred seventy-seven.

11. Three billion, fifty-seven million, four hundred seventeen thousand sixty.

ROMAN NOTATION

15. The Roman notation, instead of using figures to represent numbers, uses seven capital letters, as follows: I, V, X, L, C, D, M.

Repeat the letters in the above order until you can say them very rapidly.

The values of these letters are as follows:

I = 1 (II = 2, III = 3).

V = 5 (IV = 4, VI = 6, VII = 7, VIII = 8).

X = 10 (IX = 9, XI = 11, XII = 12, XXX = 30).

L = 50 (XL = 40, LX = 60, LXXXVIII = 88).

C = 100 (XC = 90, CCC = 300, XCIX = 99).

D = 500 (CD = 400, DCCIX = 709).

M = 1000 ($\overline{\text{M}}$ = 1000000, MDC = 600, MXVI = 1016).

To express other numbers these same letters are combined according to the following

Principles of Roman Notation

16. 1. Placing a letter after one of greater value adds its value to that of the greater.

2. Placing a letter before one of greater value subtracts its value from that of the greater.

3. Placing a letter between two letters of greater value subtracts its value from their sum.

4. Repeating a letter repeats its value.

5. Placing a bar over a letter multiplies the value of the letter by 1000.

ILLUSTRATIONS

17. 1. $X = 10$, $V = 5$, $XV = 10 + 5 = 15$. Which principle does this illustrate?

2. $V = 5$, $I = 1$, $IV = 5 - 1 = 4$. Which principle does this illustrate?

3. $C = 100$, $L = 50$, $X = 10$, $CXL = 100 + 50 - 10 = 140$. Which principle does this illustrate?

4. $X = 10$, $XXX = 10 + 10 + 10 = 30$. Which principle does this illustrate?

5. $D = 500$. $\overline{D} = 500000$. Which principle does this illustrate?

6. $CCCLX = 360$. Which principle?

7. $MCM = 1900$. Which principle?

8. $MDCLXVI = 1666$. Which principle?

9. Write numbers to illustrate all the principles of Roman notation.

10. Express in Roman notation all numbers from 1 to 100.

18. Read:

1. XLV.	6. MCIV.	11. DCXCIV.
2. CDII.	7. DCCVI.	12. MCCV.
3. MI.	8. MXIX.	13. CCXI.
4. XIX.	9. DCX.	14. MMV.
5. LXXXVI.	10. CDLI.	15. DLXVI.

Write in Roman notation:

1. 284	5. 319	9. 2,870
2. 98	6. 1,515	10. 837
3. 1,013	7. 745	11. 1,400
4. 56	8. 47	12. 245

NOTATION OF FEDERAL MONEY

19. Federal Money is the money used in the United States.

10 mills = 1 cent. Mills are not coined. The smallest coin used is the cent. Cents are written ct. or ¢.

100 cents = 1 dollar, written $1. When dollars and cents are written together, a period, called the decimal point, is placed between them. Thus, four dollars and sixty-seven cents is written $4.67.

The first two places at the right of the point are occupied by cents, the third place by mills.

Nine cents four mills is written $.094.

Four dollars sixty-five cents three mills is written $4.653.

Copy and read:

1. $ 8.40	2. $16.30	3. $ 92.003
$ 17.03	$ 9.	$ .105
$ 2.842	$ .087	$256.428
$200.504	$4007.91	$ 94.02

Write in figures:

1. Seventy dollars twenty-six cents.

ADDITION

20. Addition is the process of uniting two or more like numbers into one number. Thus, 2 and 5 are 7.

21. The numbers added are **Addends.** Thus, 3 and 10 are 13; 3 and 10 are the addends.

22. The result of addition is the **Sum.** Thus, 8 books and 7 books are 15 books; 15 is the sum.

The addends and sum are called the **terms of Addition.**

23. The **Sign of Addition** is a vertical cross placed between the addends. Thus, 5 + 9 are 14.

The sign = means *are* or *equals*. Thus, 8 + 3 = 11.

What kind of numbers can be added?

24. Add at sight:

2	6	7	3	4	5	5	9	9
5	3	1	7	2	1	6	0	5
2	3	4	5	6	7	8	9	9
1	3	4	4	1	2	5	6	8
2	3	4	5	6	7	8	9	1
2	3	8	8	4	4	3	3	9
2	3	4	5	6	7	6	8	9
7	9	8	7	5	7	8	8	7

2	3	4	5	6	7	8	9	2
3	4	5	5	2	5	4	4	6
2	3	4	9	6	7	7	8	9
4	5	6	5	6	6	5	7	2

25. Oral.

1. Ruth had 7 cents and earned 8 cents more. How many cents had she then?

2. We had 8 books in our library and bought 10 more. How many have we now?

3. Five pupils are absent from a class and 12 are present. How many pupils are there in the class?

4. Edward had 10 cents in his bank and put in 6 more. How much money has he now?

5. There are 12 pears in one box and 10 in another. How many in both boxes?

6. Albert spent 8 cents for a ball, 4 cents for a pencil, and 12 cents for a writing-book. How much did he pay for all?

7. Willard had 7 marbles in one bag and 12 in another. How many had he in both bags?

8. A boy sold a pair of skates for 20 cents, which was 10 cents less than they cost. What did they cost?

9. John paid 5 cents for paper, 2 cents for pens, and 4 cents for pencils. How many cents did he pay?

10. Will has six marbles and Frank 11. How many have both?

11. Mary bought a nine-cent ball and a twelve-cent pineapple. How much did she pay for both?

12. Susie had 12 violets and picked 9 more. How many had she then?

13. Three boys went fishing. One caught 15 fish, another 11, and the other 16. How many fish were caught?

14. A boy spent 8 cents for a slate and 21 cents for a book. How much did both cost?

15. A boy had a nickel. He found a dime, and his mother gave him 8 cents. How many cents did he then have?

26. Oral. Add:

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**	**9.**
3	5	4	5	6	7	3	7	7
2	6	8	7	8	5	0	5	4
6	8	2	9	3	8	9	2	3

10.	$6+5+4=?$	$8+5+3=?$	$7+6+8=?$
11.	$4+6+3=?$	$7+9+8=?$	$5+5+5=?$
12.	$6+3+8=?$	$5+4+3=?$	$7+7+9=?$
13.	$6+6+8=?$	$9+8+9=?$	$6+5+4=?$
14.	$10+5+0=?$	$8+9+7=?$	$7+1+8=?$
15.	$2+10+9=?$	$4+8+7=?$	$5+1+11=?$

16. A grocer sold a melon for 25 cents and a basket of grapes for 15 cents. How much did he receive for both?

17. A boy is 7 years old. His sister is 4 years older, and his father 25 years older than his sister. How old is his father?

18. Mr. A lives 10 miles east of a village, and Mr. B 17 miles west. How many miles from Mr. A's to Mr. B's?

19. Mary had 15 cents, and John 25 cents. How many had both?

20. A drover bought 8 cows, 5 horses, and 10 sheep. How many animals did he buy?

21. Fred paid 10 dollars for a goat and 12 dollars for a cart. How much did both cost him?

27. Written. Add:

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**
3	7	8	7	6	7	8	3
4	6	9	4	0	5	5	9
5	4	2	3	9	4	7	8
9	3	5	8	5	6	6	7

In adding columns like the 1st, say 9—14—18—21, not 9 and 5 are 14 and 4 are 18, etc.

9.	**10.**	**11.**	**12.**	**13.**	**14.**	**15.**	**16.**	**17.**
7	6	4	5	4	7	8	9	5
5	5	5	9	3	5	7	7	4
3	4	6	8	9	6	5	6	3
0	9	9	6	2	2	0	4	9
6	3	8	5	7	3	7	2	8

18.	**19.**	**20.**	**21.**	**22.**	**23.**	**24.**	**25.**	**26.**
3	9	2	8	2	6	2	3	5
9	7	8	5	9	9	9	5	5
8	5	7	0	6	5	7	7	8
5	3	9	7	5	8	6	0	7
7	2	5	4	8	7	8	8	7
4	0	7	3	4	6	4	7	6
2	4	2	2	7	2	9	6	3

28.

1. $13+5$	**6.** $13+6$	**11.** $14+7$	**16.** $14+8$
2. $23+5$	**7.** $23+6$	**12.** $24+7$	**17.** $24+8$
3. $33+5$	**8.** $33+6$	**13.** $34+7$	**18.** $34+8$
4. $43+5$	**9.** $43+6$	**14.** $44+7$	**19.** $44+8$
5. $53+5$	**10.** $53+6$	**15.** $54+7$	**20.** $54+8$

21. A boy rode his wheel 23 miles on Monday, 10 miles on Tuesday, and 5 miles on Wednesday. How many miles did he ride in the three days?

22. A farmer purchased a cow for 35 dollars, and a pig for 8 dollars. What was the cost of both?

23. In a certain school 28 pupils were present, 5 were absent on account of sickness, and 4 were absent for other reasons. How many pupils belonged to the school?

24. A farmer sold 25 bushels of apples to one man, 10 to another, and 8 to another. How many bushels did he sell?

25. John had 40 cents in his bank. He added 8 cents on Monday, and 10 cents on Wednesday. How much money had he then in his bank?

26. A man paid for paint 56 dollars, and for labor 10 dollars. How much did he pay for both?

27. Bought sheep for 50 dollars, turkeys for 12 dollars, and chickens for 8 dollars. How much did they cost?

28. A man in repairing his house paid 35 dollars for lumber, 8 dollars for paint, 2 dollars for nails, and 10 dollars for labor. What was the cost of his repairs?

29. How many fish did Mr. A catch in 4 days if he caught 12 the first day, 8 the second, 9 the third, and 7 the fourth?

30. A girl spent for car fare 5 cents, for pencils 4 cents, for paper 8 cents, for ribbon 10 cents, for lunch 15 cents, and had 9 cents left. How much money had she at first?

31. John has 20 marbles, Henry 4 more than John, and Fred 5 more than Henry. How many marbles has Fred?

Add:

32. 42 9	**33.** 52 9	**34.** 62 9	**35.** 72 9	**36.** 82 9	**37.** 92 9
38. 34 8	**39.** 54 8	**40.** 74 8	**41.** 94 8	**42.** 44 8	**43.** 64 8
44. 28 7	**45.** 48 7	**46.** 68 7	**47.** 38 7	**48.** 58 7	**49.** 78 7

50.	51.	52.	53.	54.	55.
24	34	54	74	44	64
8	8	8	8	8	8
8	8	8	8	8	8

29. Written.—1. Add 4150
and 5827

7 units and 0 units are how many units?
2 tens and 5 tens are how many tens?
8 hundreds and 1 hundred are how many hundreds?
5 thousands and 4 thousands are how many thousands?
All the above work might be expressed thus:

$$\left.\begin{array}{r} 4150 \\ +\ 5827 \end{array}\right\} \text{Addends.}$$
$$= 9977 \quad \text{Sum.}$$

2. Three or more numbers may be added in the same way, thus:

$$\left.\begin{array}{r} 543{,}122 \\ 12{,}367 \\ 10{,}200 \\ 1{,}100 \end{array}\right\} \text{Addends.}$$
$$566{,}789 \quad \text{Sum.}$$

3. Add:

763
528
659
435

How should we write the addends so that we may easily add units and units, tens and tens, etc.?

5 units + 9 units + 8 units + 3 units are how many units?
25 units make how many tens and how many units over? Put the 5 units under units' column, and add 2 to tens' column.

2 tens + 3 tens + 5 tens + 2 tens + 6 tens are how many tens? 18 tens make how many hundreds and how many tens over?

Put the 8 tens under tens' column, and add 1 to hundreds' column. 1 hundred + 4 hundreds + 6 hundreds + 5 hundreds + 7 hundreds are how many hundreds? 23 hundreds make how many thousand and how many hundreds over?

Put 3 hundreds under hundreds' column and 2 thousands in thousands' column in the sum. What would you do with 2 thousands if there were thousands in the addends?

30. From the foregoing examples we may make the following

Rule for Addition

1. Write the addends so that the figures in units' place in all the addends shall stand in the same column.

2. Add the figures in the right-hand column. If the sum is less than ten, write it under units' column. If it is ten or more, divide it by ten, write the remainder under units' column, and add the quotient with the figures in tens' column. Proceed in this way till all the columns have been added.

3. Whenever the sum of any column is 10 or more, divide it by 10, put the remainder under the column added, and add the quotient with the next column to the left.

4. To add Federal Money, write the addends so that all the decimal points shall stand in the same column. Add the same as other numbers, and place the decimal point in the sum under the decimal points in the addends.

Proof of Addition

1. Add the numbers in a different order. If the results agree, the work is generally correct. Or,

2. Subtract the addends, one at a time, from the sum. If the last result is zero, the addition is correct.

31. Add:

4.	**5.**	**6.**	**7.**	**8.**	**9.**
36	25	96	25	87	38
70	98	39	75	96	49
21	76	78	84	48	96
84	29	54	26	93	44

10.	**11.**	**12.**	**13.**	**14.**
28	639	1050	126	$115.85
39	874	394	149	327.15
76	596	769	1260	495.27
42	421	564	1004	160.03
89	397	285	986	598.09
73	269	784	24	784.06

15.	**16.**	**17.**	**18.**	**19.**
97	857	283	$208.40	$356.24
98	943	2075	32.03	35.09
79	268	298	26.07	2.15
68	207	963	18.94	30.05
40	976	859	236.29	5.16
87	888	876	28.15	304.29

20.	**21.**	**22.**	**23.**
2673	837	628	8063
846	2964	4307	259
1025	418	526	8264
92	3825	8279	1287
837	842	428	428
642	29	4273	3064

Add:

24.	25.	26.	27.
$26.43	$715.30	$165.00	$ 20.863
18.75	21.86	8.429	129.40
2.93	9.246	113.82	5208.00
4.10	10.163	6.804	.926
128.06	7.208	39.625	128.753
563.13	516.00	11.31	37.15
28.00	8.096	476.203	192.097

28. Add seventeen thousand, nine hundred six; four thousand, two hundred eighty-nine; eight hundred twelve thousand, seven hundred eight; six hundred two; forty-two thousand, nine hundred two; twelve thousand.

29. Find the sum of eleven thousand, six hundred seventeen; sixty-eight; four thousand, twenty-five; two thousand, three hundred nine; eighty-five thousand.

30. A merchant's sales were $2963.84 in January, $1463.27 in February, $3846.25 in March, and $2016.92 in April. What did his sales amount to in the four months?

31. Find the sum of all the numbers between and including 167 and 174.

32. A man had $170 in his pocket, which was $70.75 less than he had in his safe. How much money had he in the safe?

33. Add nine dollars six cents; fifteen dollars seventy-two cents; sixty dollars eighty-seven cents; fifty-nine cents; four dollars five cents; two hundred dollars thirteen cents.

34. Add 3160, 4980, 7592, 8324, 6958.

35. Add 21563, 74321, 58026, 79532, 43984.

36. Add 32162, 19073, 76352, 698725, 49623.

37. Add 36, 512, 7198, 41960, 75, 34, 812, 916.

38. Add $5.28, $13.936, $17.84, $16.05, $28.006.

39. Add $28.75, $19.87, $16.97, $14.82, $17.64, $18.26, $19.78.

40. Add 28, 97, 386, 9428, 359601, 4289, 86.

41. 69872, 594760, 653051, 21876, 394321.

42. Add 8 thousand 312; 25 thousand 906; 215 thousand 7 hundred 8; 15 thousand 9 hundred 15; 56 thousand 2 hundred 74; 328 thousand 906.

43. Add 5 dollars 18 cents 5 mills; 28 dollars 1 cent; 19 dollars 24 cents 5 mills; 17 dollars 18 cents; 84 dollars 2 cents 9 mills; 29 dollars 8 cents; 328 dollars 50 cents 4 mills.

PROBLEMS IN ADDITION

32. **1.** Georgie has 18 peaches, and Donald 23 peaches. How many have both?

2. In a schoolroom are 24 boys and 17 girls. How many children are there in all?

3. A farmer has 21 sheep in one pasture, 32 in another, and 45 in a third. How many sheep has he in all?

4. A boy picked up 26 bushels of potatoes on Monday, 29 bushels on Tuesday, and 32 bushels on Wednesday. How many bushels did he pick up in all?

5. Frank paid 35 cents for a knife, 45 cents for a hammer, and 50 cents for a saw. How much did he pay for all?

6. Mr. Smith has 42 apple trees, 24 pear trees, 16 plum trees, and 19 cherry trees. How many fruit trees has he?

7. John picked 23 quarts of berries, Henry 19 quarts, Sarah 26 quarts, and Mildred 12 quarts. How many quarts did they all pick?

8. Charles lives 84 miles west of Chicago, and William 49 miles east. How far apart do they live?

9. In a yard are 56 hens, 24 ducks, 28 turkeys, and 18 geese. How many fowls in the yard?

10. A lady bought eggs for 45 cents, sugar for 75 cents, tea for 68 cents, and meat for 32 cents. How much did she pay for all?

11. Mary is 10 years old, Clara is 5 years older than Mary, and their brother is as old as both of them together. What is the sum of their ages?

12. On one side of a street are 62 houses, and on the other 59 houses. How many houses are on the street?

13. If a boy is 10 minutes late at school on Monday, 8 minutes on Tuesday, 15 minutes on Wednesday, 7 minutes on Thursday, and 12 minutes on Friday, how many minutes does he lose in the week?

14. Three boys went fishing, and caught 16 perch, 19 pickerel, and 8 black bass. How many fish did they catch in all?

15. Two trains starting from the same place run two days in opposite directions. One runs 530 miles the first day and 525 miles the second, while the other runs 492 miles the first day and 510 miles the second. How far apart are they at the end of the two days?

16. A man bought coal for $5.60, wood for $3.45, and a stove for $45. What was the whole cost?

17. There are 112 bushels of wheat in one bin, 175 in another, and 234 in the third. How many bushels in all?

18. There are 218 pages in my reader, 245 pages in my arithmetic, and 195 pages in my geography. How many pages in the three books?

SUBTRACTION

33. **Subtraction** is the process of finding the difference between two like numbers. Thus, 21 less 7 = 14; 13 ct. less 5 ct. = 8 ct.

34. The number from which we subtract is the **Minuend.** Thus, 15 less 9 = 6; 15 is the minuend. The number subtracted is the **Subtrahend.** Thus, 12 cents less 5 cents = 7 cents; 5 cents is the subtrahend.

35. The result of subtraction is called the **Difference** or **Remainder.** Thus, 25 miles less 15 miles = 10 miles; 10 miles is the difference or remainder. The minuend, subtrahend, and remainder are called the **Terms of Subtraction.**

The **Sign of Subtraction** is a short horizontal line placed before the subtrahend; thus, 12 birds − 4 birds = 6 birds.

What kind of numbers can be subtracted? How does the minuend compare with the subtrahend? With the remainder?

36. Subtract at sight:

1. 14 − 4	**11.** 12 − 5	**21.** 16 − 10	**31.** 8 − 6
2. 12 − 7	**12.** 16 − 8	**22.** 18 − 8	**32.** 7 − 1
3. 15 − 9	**13.** 13 − 6	**23.** 19 − 5	**33.** 9 − 0
4. 17 − 8	**14.** 11 − 8	**24.** 7 − 6	**34.** 11 − 3
5. 9 − 5	**15.** 16 − 5	**25.** 10 − 2	**35.** 10 − 6
6. 11 − 4	**16.** 7 − 4	**26.** 11 − 10	**36.** 15 − 9
7. 13 − 3	**17.** 11 − 6	**27.** 15 − 15	**37.** 20 − 10
8. 7 − 7	**18.** 9 − 2	**28.** 13 − 2	**38.** 13 − 7
9. 15 − 5	**19.** 8 − 5	**29.** 14 − 7	**39.** 15 − 3
10. 17 − 10	**20.** 7 − 3	**30.** 16 − 2	**40.** 14 − 3

41.	$7 + ? = 14$	**51.**	$15 + ? = 20$	**61.**	$18 - 9 = ?$
42.	$8 + ? = 11$	**52.**	$11 + ? = 15$	**62.**	$13 - 11 = ?$
43.	$6 + ? = 15$	**53.**	$9 + ? = 14$	**63.**	$15 - 10 = ?$
44.	$5 + ? = 11$	**54.**	$8 + ? = 15$	**64.**	$17 - 10 = ?$
45.	$7 + ? = 16$	**55.**	$7 + ? = 12$	**65.**	$? + 5 = 11$
46.	$8 + ? = 16$	**56.**	$8 + ? = 11$	**66.**	$? + 7 = 16$
47.	$6 + ? = 9$	**57.**	$17 - 8 = ?$	**67.**	$? + 5 = 16$
48.	$7 + ? = 15$	**58.**	$14 - 5 = ?$	**68.**	$? + 6 = 19$
49.	$11 + ? = 14$	**59.**	$17 - 5 = ?$	**69.**	$? + 4 = 13$
50.	$13 + ? = 16$	**60.**	$20 - 9 = ?$	**70.**	$? + 6 = 11$

37. Oral.

1. Frank had 15 cents in his bank and took out 5 cents. How many cents remained?

2. There were 13 sparrows upon a limb, and 5 flew away. How many sparrows remained on the limb?

3. A farmer planted potatoes and corn in a field containing 12 acres. He planted potatoes in 7 acres. In how many acres was corn planted?

4. John had 14 marbles and gave away 6 of them. How many had he left?

5. Frank lives 12 blocks east of the schoolhouse, and Henry 5 blocks east. How many blocks between Frank's home and Henry's home?

6. In a garden there are 20 rose bushes. Only 10 of them bear red roses. How many bushes do not bear red roses?

7. Mary added two numbers, and her answer was 18. The smaller number was 8. What was the larger?

8. A farmer having 17 cows sold 6 of them. How many remained?

9. A clerk earns 14 dollars a week and spends 6 dollars. How much does he save each week?

10. Mary bought ribbon for 10 cents and paper for 5 cents. How much had she remaining from 20 cents?

11. From a basket containing 16 oranges, 7 were taken. How many were left?

12. James is 7 years younger than his brother, who is 15 years old. How old is James?

13. If you have 19 cents and spend 10 cents, how much have you left?

14. A boy sold for 12 cents a ball that cost him 20 cents. How much did he lose?

15. How much more is 20 than 11?

16. Tom has 20 marbles, and Edward 11. How many has Tom more than Edward?

17. If you give 15 cents in payment for a slate worth 20 cents, how much do you still owe?

18. Nell had 21 chickens, but a dog killed 10 of them. How many were left?

19. Lucy is 20 years old, and her sister 6 years younger. How old is her sister?

20. John had $20. He spent $10 for a coat and $3 for a hat. How much had he left?

21. A pole is 19 feet long. I cut off 4 feet at one time and 6 feet at another. How many feet were left?

38. Oral. Subtract:

1. 38 − 7	**2.** 48 − 7	**3.** 58 − 7	**4.** 88 − 7	**5.** 68 − 7	**6.** 78 − 7	**7.** 98 − 7
8. 13 − 5	**9.** 23 − 5	**10.** 53 − 5	**11.** 73 − 5	**12.** 83 − 5	**13.** 63 − 5	**14.** 43 − 5
15. 26 − 8	**16.** 36 − 8	**17.** 56 − 8	**18.** 66 − 8	**19.** 46 − 8	**20.** 76 − 8	**21.** 96 − 8

22.	21	**23.**	31	**24.**	41	**25.**	71	**26.**	91	**27.**	51	**28.**	81
	9		9		9		9		9		9		9

29.	$15 - 3$	**33.**	$55 - 3$	**37.**	$14 - 7$	**41.**	$54 - 7$
30.	$25 - 3$	**34.**	$65 - 3$	**38.**	$24 - 7$	**42.**	$64 - 7$
31.	$35 - 3$	**35.**	$75 - 3$	**39.**	$34 - 7$	**43.**	$74 - 7$
32.	$45 - 3$	**36.**	$85 - 3$	**40.**	$44 - 7$	**44.**	$84 - 7$

45. Subtract 6 from each of these numbers:

15, 25, 35, 45, 55, 65, 75, 85, 95.

46. $3 + 6 + 5 + 8 - 10 = ?$ $8 + 6 + 5 + 5 - 8 = ?$

47. There are 45 pupils in our class. 8 are absent. How many are present?

48. A man earns $56 a month. He pays $7 a month for rent. How much is left?

49. 51 birds were on a tree. 7 flew away. How many remained?

50. A lady had 54 pounds of butter in a tub, and used 7 pounds. How many pounds remained in the tub?

51. Mr. R. had 42 animals in a field. 5 of them were horses, and the rest cows. How many were cows?

52. Ella had 48 cents. She gave 5 to Mary and 4 to Lucy. How many had she left?

NOTE. — Subtract 5 from 48, then 4 from the result.

53. Will had 53 marbles and gave 9 to Max and 8 to Fred. How many had he left?

39. 1. From 9867
take 4263

7 units less 3 units are how many units? Write 4 units under units' column.

6 tens − 6 tens = how many tens? Write 0 under tens' column.

8 hundreds − 2 hundreds = how many hundreds? Write 6 hundreds under hundreds' column.

9 thousands − 4 thousands = how many thousands? Write 5 thousands under thousands' column.

All this work may be expressed thus:

9867 Minuend.
− 4263 Subtrahend.
= 5604 Remainder.

2. From 4285
take 597

From 5 units take 7 units. Can it be done? From 8 tens take 1 ten and change the 1 ten to units. It equals how many units? Add 10 units to 5 units. How many units does it make? From 15 units take 7 units. How many units are left? Write 8 units under units' column.

Since you have taken 1 ten from 8 tens in the minuend, how many tens are left? From 7 tens take 9 tens. Can it be done? From 2 hundreds take 1 hundred and change it to tens. 1 hundred = how many tens? Add 10 tens to 7 tens. How many tens does it make? From 17 tens take 9 tens. How many tens are left? Write 8 tens under tens' column.

Since you have taken 1 hundred from 2 hundreds, how many hundreds are left in the minuend? From 1 hundred take 5 hundreds. Can it be done? From 4 thousands take 1 thousand. One thousand = how many hundreds? Add 10 hundreds to 1 hundred. How many hundreds does it make? From 11 hundreds take 5 hundreds. How many hundreds are left? Write 6 hundreds under hundreds' column.

Since you have taken 1 thousand from 4 thousands, how many thousands are left? Write 3 thousands in thousands' place in the remainder.

All this work may be expressed thus:

$$\begin{array}{rl} & 4285 \text{ Minuend.} \\ - & 597 \text{ Subtrahend.} \\ = & 3688 \text{ Remainder.} \end{array}$$

40. From the examples given above we may make the following

Rule for Subtraction

1. Write the subtrahend under the minuend so that units shall come under units.

2. From units take units, from tens take tens, and so on till each figure of the subtrahend has been subtracted from the figure above it, writing the remainder, every time, below the figures subtracted.

3. Whenever a figure of the minuend is less than the figure of the subtrahend below it, take 1 from the next figure to the left in the minuend, and add 10 to the figure which is too small. Then subtract the subtrahend figure from the sum obtained.

4. To subtract Federal money, write the subtrahend under the minuend so that the decimal point of the subtrahend is under the decimal point of the minuend. Then subtract the same as other numbers.

41. 1. $12 - ? = 7$? Which terms of subtraction are given in this question? Which term are you asked to find? How do you find it? When the minuend and remainder are given, how can the subtrahend be found?

2. $? - 5 = 7$? Which terms are given? Which term are you to find? How do you find it? When the subtrahend and remainder are given, how can the minuend be found?

42. The answers to these questions give us the following

Proof of Subtraction

1. Subtract the remainder from the minuend. If the result is the same as the subtrahend, the subtraction is correct. Or,

2. Add the subtrahend and remainder. If the sum is the same as the minuend, the subtraction is correct.

43. Subtract:

1. 2819 674	**2.** 8203 1276	**3.** 4295 597	**4.** 7306 1807
5. 2763 1289	**6.** 37284 9287	**7.** 36801 18463	**8.** 18003 921
9. 92874 11392	**10.** 94210 8206	**11.** 42840 38706	**12.** 98301 26942
13. 38264 29842	**14.** 19327 8291	**15.** 92593 87246	**16.** 27075 18092
17. $2.156 1.124	**18.** $35.283 17.05	**19.** 25.18 1.155	**20.** $36.514 16.827
21. $34.28 24.28	**22.** $39.216 27.134	**23.** $17.804 16.752	**24.** $75.005 24.325

25. From seventeen thousand sixteen, take nine thousand four hundred eighty-seven.

26. From seventy-two thousand three hundred eleven, take forty-six thousand nine hundred sixty-one.

27. Take eight thousand four, from thirty thousand.

28. Find the difference between sixteen thousand four hundred seventy-five, and twenty thousand seven hundred twenty-seven.

29. The minuend is 7026 and the subtrahend 987. Find the difference.

30. Find the difference between 9284 and 13,411.

31. $\$4216.56 - \$1874.92 = ?$

32. $\$149.725 - \$69.417 = ?$

33. Take sixteen thousand eight hundred seventeen, from twenty-four thousand five hundred forty-one.

34. The difference is 8037. The minuend is 19,406. Find the subtrahend.

35. Take seventy-six dollars four cents two mills, from one hundred two dollars nine mills.

36. From sixty-five thousand three hundred sixteen, take twenty-four thousand forty.

37. Take $\$86.215$ from $\$900.09$.

38. What must be added to $\$.67$ to make $\$3$?

39. From a farm of 263 acres of land 97 acres were sold. How much was left?

40. A man had $\$279$, and spent $\$129.64$. How much had he left?

41. From a cask of vinegar, containing 44 gallons, 17 gallons leaked out. How many gallons remained?

42. From a box containing 200 oranges, 127 were sold. How many oranges remained?

43. A farmer having 250 acres of land, sold 87 acres. How many acres did he have left?

44. Two boys start together and run in the same direction. How far apart are they when one has run 192 yards and the other 156 yards?

45. There were 83 sparrows on a limb, and 34 flew away. How many remained?

46. A boy having $2 spent $.75 for a pair of skates. How much had he left?

47. Genevieve had $1, and gave Edith 37 cents. How much had she left?

48. There are 49 eggs in a basket. How many more should be put in to make 144 eggs in all?

49. John picked 40 quarts of berries, and James picked 13 quarts less than John. How many quarts did James pick?

REVIEW PROBLEMS

ADDITION AND SUBTRACTION

44. 1. A farmer, having 456 bushels of corn, sold 84 bushels to one man and 135 bushels to another. How many bushels did he have left?

2. A man started to walk 112 miles in three days. He walked 32 miles the first day, and 41 miles the second. How far must he walk the third day to complete the journey?

3. I bought a cow for $42, another for $48, and a third for $56. For how much should I sell them to gain $28?

4. A lady bought sugar for 65 cents, tea for 55 cents, molasses for 72 cents, butter for 84 cents, starch for 25 cents, and gave in payment a five-dollar bill. How much change should she receive?

5. John's father gave him $2.25, and his uncle gave him $1.40. He earned enough besides so that he bought, with the whole, a suit of clothes for $8. How much did he earn?

6. Two vessels start from points 850 miles apart, and sail toward each other. How far are they apart when one has sailed 246 miles and the other 352 miles?

7. A man sold one horse for $145 and another for $182. On the first he gained $23, and on the second $36. What was the cost of both?

8. A boy bought apples for \$.45 and pears for \$.62, and sold them all for \$1.50. What was his profit?

9. John sold 62 newspapers, Frank 48, and Henry 27 less than both of them. How many did Henry sell?

10. A grocer sold butter for \$45 and cheese for \$62. On the butter he lost \$6 and on the cheese he gained \$14. What was the cost of both?

11. In a school of 480 pupils, 13 were absent from the primary department, 11 from the junior, and 9 from the senior. How many were present in the whole school?

12. A man earning \$800 a year paid \$208 for board, \$175 for clothes, and \$266 for other purposes. What does he save?

13. Mr. Clark's age, which is 50 years, is 17 years more than the sum of the ages of his son and daughter. His daughter is 18 years old. How old is his son?

14. A farmer bought a barrel of flour for \$6.35, sugar for \$2.15, coffee for \$1.46, tea for \$1.20, and gave in payment \$3.15 worth of butter and the remainder in cash. What did he pay in money?

15. The sum of 52 and 64 is how much greater than the difference between 124 and 69?

16. From a flock of 320 sheep were sold at one time 76 and at another 112. How many remained?

17. A man bought 148 bushels of potatoes of A, 216 bushels of B, 183 bushels of C, and afterwards sold all but 137 bushels. How many bushels did he sell?

18. The sum of three numbers is 342. Two of the numbers are 84 and 96. What is the third number?

19. A farmer having 215 acres of land, used 21 acres for corn, 36 for oats, 29 for barley, 18 for potatoes, 52 for meadow, and the rest for pasture. How many acres were used for pasture?

MULTIPLICATION

45. **Multiplication** is finding a number which is a certain number of times another number. Thus, 6 times 5 dollars = 30 dollars.

46. The number multiplied is the **Multiplicand.** Thus, 7 times 9 = 63. 9 is the multiplicand.

47. The number by which we multiply is the **Multiplier.** Thus, 12 times 7 = 84. 12 is the multiplier.

48. The result of multiplication is the **Product.** Thus, 6 times 3 feet = 18 feet. 18 feet is the product.

49. The multiplicand, multiplier, and product are called the **Terms of Multiplication.**

50. The **Sign of Multiplication** is an oblique cross placed after the multiplicand. Thus, 12×6 means 6 times $12.

51. A number that is applied to some particular things or objects is a **Concrete Number.** Thus, 10 cows, 5 books, 6 gallons.

52. A number that is not applied to anything is an **Abstract Number.** Thus, 10, 5, 6.

53. The product is always the same kind of a number as the multiplicand. Thus, 12 pounds $\times$ 3 = 36 pounds. Both the multiplicand and product are pounds.

54. *The multiplier is always regarded as an abstract number.*

55. Tell products at sight:

3	4	5	9	8	4	6	5	8
6	7	8	6	5	8	8	4	3
9	5	6	3	6	7	4	0	7
2	6	4	7	6	6	0	4	10
9	3	4	10	10	11	11	12	12
8	7	10	2	7	2	7	2	7
3	5	4	5	9	7	3	4	10
8	8	5	3	9	7	5	11	3
10	11	11	12	12	9	5	4	6
8	3	8	3	8	3	9	6	4
9	3	4	10	10	11	11	12	12
10	9	12	4	9	4	9	4	9
10	10	4	6	9	4	10	10	11
3	5	7	6	10	8	5	10	5
11	11	10	12	12	8	5	8	3
10	5	11	5	10	7	9	2	7
9	3	4	8	9	5	12	4	12
12	12	7	8	9	5	12	4	11

56. Oral.

1. Multiply these numbers by 6. By 8. By 9:

7, 9, 4, 3, 8, 11, 5, 10, 12, 2, 6.

2. At 3 cents each, what will 5 oranges cost?

3. Fred saves 8 cents a day. How much can he save in 9 days?

4. At $5 each, what will 11 sheep cost?

5. Eight five-cent tablets cost how much?

6. How many trees must I plant to make a square orchard with 7 trees on a side?

7. At $4 a week, how much will a man's board bill amount to in 11 weeks?

8. There are 8 pints in a gallon. How many pints in 9 gallons?

9. There are 4 pecks in a bushel. How many pecks in 6 bushels.

10. One horse eats 6 quarts of oats a day. How many quarts will 10 horses eat in a day?

11. What will a dozen oranges cost at 3 cents each?

12. A man can build 5 rods of fence in a day. How many rods can he build in 12 days?

13. Cloves cost 7 cents an ounce. What will 8 ounces cost?

14. How many quarts of water will be contained in 6 8-quart pails?

15. On a card are 6 rows of buttons with 10 buttons in a row. How many buttons on the card?

16. James has 7 marbles, and Henry 6 times as many. How many marbles has Henry?

17. At 5 cents a quart, what is my milk bill for 7 days, if I use 2 quarts a day?

18. A steamer travels 12 miles an hour. How far can she travel in 6 hours?

19. There are 4 pecks in a bushel. How many pecks in 12 bushels?

20. John has 2 cents, Robert twice as many as John, and Will 3 times as many as Robert. How many cents has Will?

21. Ten mills make one cent. How many mills in 10 cents?

22. At $.05 a quart, what will be the cost of 9 quarts of bird seed?

23. At 12 cents a quart, what will 10 quarts of molasses cost?

57. Multiply 4127
by 6

6 times 7 units are how many units? 42 units equal how many tens and how many units over? Write 2 units in units' place in the product, and add the 4 tens to the product of tens.

6 times 2 tens are how many tens? 12 tens and 4 tens are how many tens? 16 tens equal how many hundreds and how many tens over? Write 6 tens in tens' place in the product, and add 1 hundred to the product of hundreds.

6 times 1 hundred are how many hundreds? 6 hundreds and 1 hundred are how many hundreds? Write 7 hundreds in hundreds' place in the product.

6 times 4 thousands are how many thousands? 24 thousands = how many ten-thousands and how many thousands over? Write 4 thousands in thousands' place and 2 ten-thousands in ten-thousands' place in the product.

The work may be expressed thus:

$$\begin{array}{r} 4127 \text{ Multiplicand.} \\ \times\ 6 \text{ Multiplier.} \\ \hline 24762 \text{ Product.} \end{array}$$

58. Written.

Find the products:

1. 45 × 5	**2.** 36 × 4	**3.** 82 × 7	**4.** 63 × 6
5. 72 × 9	**6.** 84 × 5	**7.** 43 × 9	**8.** 35 × 8
9. 75 × 6	**10.** 98 × 9	**11.** 73 × 10	**12.** 68 × 11
13. \$123 × 9	**14.** \$236 × 5	**15.** \$384 × 8	**16.** \$721 × 9
17. \$398 × 10	**18.** \$4.32 × 5	**19.** \$2.36 × 8	**20.** \$3.95 × 4
21. 387 × 12	**22.** 792 × 7	**23.** 684 × 4	**24.** 199 × 12
25. 1306 × 5	**26.** 2908 × 9	**27.** 4908 × 6	**28.** 3967 × 7
29. 3872 × 8	**30.** 3963 × 11	**31.** 5984 × 7	**32.** 7983 × 10
33. 3161 × 11	**34.** 5184 × 9	**35.** 3872 × 5	**36.** 4560 × 8

Multiply these numbers by 2, 5, 6, 9, 8, 7:

37. 42,608	**39.** 29,132	**41.** 18,736	**43.** 32,984
38. 36,975	**40.** 13,671	**42.** 28,416	**44.** 15,116

59. 1. Write 536. In what place is the 6? The 3? The 5? Annex a cipher to 536. Thus, 5360.

In 5360, what places do the 6, 3, and 5 occupy? What has been done to each figure?

Moving a figure one place to the left affects its value how? (See principles of Arabic notation.) Since the value of each figure has been multiplied by 10, what has been done to the entire number?

Since annexing a cipher to a number multiplies it by 10, what would you do to multiply a number by ten?

2. Multiply these numbers by 10:

25, 342, 607, 4936, 8972, 400, 91.

How can you multiply a number by 100? By 1000? By 10000?

3. Multiply the following numbers by 100, 1000, 10000:

29, 341, 256, 9203, 4216, 899.

4. Multiply 876 by 80.

$80 = 8 \times 10$. $876 \times 80 = 876 \times 8 \times 10$

First multiply 876 by 8, then by 10, thus:

876
80 How did we multiply by 10?
70080 Product.

5. Multiply 876 by 800.

$800 = 8 \times 100$. $876 \times 800 = 876 \times 8 \times 100$, thus:

876
800 How did we multiply by 100?
700800 Product.

How could you multiply by 7000?

6. Multiply 1283 by 967.

$967 = 900 + 60 + 7$.

Multiply 1283 by 7, by 60, by 900, and add the three products, thus:

```
   1283
    967
   8981
  76980
1154700
1,240,661  Product.
```

The ciphers at the right of the partial products may be omitted, thus:

```
   1283
    967
   8981
   7698
  11547
1,240,661  Product.
```

60. From the above examples we may make the following

Rule for Multiplication

1. Write the multiplier under the multiplicand.

2. Multiply the units in the multiplicand by the multiplier. If this product is less than 10, write it in units' place in the product. If 10 or greater than 10, divide it by 10 and write the remainder in units' place in the product, adding the quotient to the product of the tens. Proceed in a similar way toward the left till all the figures of the multiplicand have been multiplied.

3. If the multiplier contains more than one figure, multiply the multiplicand by each figure of the multiplier separately, writing the first figure of each partial product under the figure multiplied by, and add the partial products.

4. In multiplying Federal money put the decimal point in the product under the decimal point in the multiplicand.

61. Multiply

1. 324×24	**12.** 598×36	**23.** $\$36.915 \times 45$
2. 296×39	**13.** 287×49	**24.** $\$126.93 \times 87$
3. 387×45	**14.** 799×99	**25.** $\$17.856 \times 48$
4. 263×56	**15.** 296×28	**26.** $\$19.634 \times 49$
5. 892×63	**16.** 694×39	**27.** $\$75.105 \times 97$
6. 728×75	**17.** 206×54	**28.** $\$16.351 \times 64$
7. 398×84	**18.** $\$28.15 \times 28$	**29.** $\$280.52 \times 36$
8. 987×98	**19.** $\$34.98 \times 27$	**30.** $\$356.04 \times 28$
9. 516×31	**20.** $\$19.845 \times 46$	**31.** $\$987.62 \times 75$
10. 798×43	**21.** $\$7.852 \times 124$	**32.** $\$396.41 \times 41$
11. 896×79	**22.** $\$28.754 \times 15$	**33.** $\$806.04 \times 79$

62. **1.** $31 \times 10 \times 200 = ?$

2. A has 172 sheep, and B 3 times as many. How many has B?

3. There are 56 pounds in one bushel of salt. How much will 7 bushels weigh?

4. What will 6 acres of land cost, at \$84 an acre?

5. Find the cost of 8 bushels of potatoes at \$.45 a bushel.

6. There are 320 rods in 1 mile. How many rods in 9 miles?

7. How many yards in 5 miles, there being 1760 yards in one mile?

8. At \$1.75 a bushel, what will 10 bushels of walnuts cost?

9. If a train travels at the rate of 53 miles an hour, how far will it travel in 11 hours?

10. In an orchard there are 32 peach trees in a row, and 12 rows. How many trees in the orchard?

11. At $2.50 a day, what will a man earn in 6 days?

12. Find the cost of 9 yards of silk at $2.15 a yard.

13. How many pounds in 4 barrels of flour, there being 196 pounds in one barrel?

14. If 1 sack of flour is worth $1.35, what must be paid for 6 sacks of flour?

15. At $5.25 a ton, what will 12 tons of coal cost?

16. Find the cost of 10 tons of hay at $10.50 a ton.

17. There are 5280 feet in 1 mile. How many feet are there in 7 miles?

18. There are 2000 pounds in one ton. How many pounds are there in 10 tons?

19. If it costs $384 to pay the hands in a certain factory for one day, what will be the amount of the weekly pay-roll?

20. What must be paid for 86 quarts of berries at 12 cents a quart?

21. The yearly expenses of a certain family are $642. What are the expenses of a family using 3 times as much?

22. If the average daily attendance at a certain school is 563, what is the attendance for 10 days?

23. Mr. Smith has 174 acres of land, and his neighbor 5 times as much. How many acres has his neighbor?

24. If a cord of wood costs $4.50, what will 11 cords cost?

25. There are 144 pens in 1 gross. How many pens are there in 12 gross?

26. How many pounds in 8 barrels of sugar, if each barrel weighs 327 pounds?

DIVISION

63. **Division** is the process of finding how many times one number is contained in another, or finding one of the equal parts of a number. Thus, 5 cents is contained in 20 cents 4 times. One-fourth of 20 cents is five cents.

64. The number divided is the **Dividend.** The number by which another number is divided is the **Divisor.**

65. The result of division is the **Quotient.**

Thus, 45 (dividend) divided by 9 (divisor) = 5 (quotient).

The dividend, divisor, and quotient are called the **Terms of Division.**

66. When the divisor is not exactly contained in the dividend, the part of the dividend that is left is called the remainder. Thus, 59 divided by 8 = 7 and 3 remainder.

67. The **Sign of Division** is a short horizontal line with a dot above and a dot below it. When placed between two numbers, it shows that the first is to be divided by the second. Thus, $80 \div 10 = 8$.

68. Division is also indicated by placing the dividend above and the divisor below a horizontal line. Thus, $\frac{12}{3}$ means $12 \div 3$; or $\frac{1}{3}$ means $1 \div 3$, or 1 whole thing divided into 3 equal parts.

69. Tell quotients at sight:

$4 \times ? = 8$	2 in $14 = ?$	$? \times 2 = 12$
$6 \times ? = 18$	4 in $48 = ?$	$? \times 5 = 45$
$7 \times ? = 28$	12 in $84 = ?$	$? \times 6 = 30$
$4 \times ? = 36$	6 in $36 = ?$	$? \times 4 = 36$
$2 \times ? = 24$	9 in $45 = ?$	$? \times 8 = 40$
$3 \times ? = 33$	4 in $44 = ?$	$? \times 7 = 63$
$6 \times ? = 24$	12 in $60 = ?$	$? \times 12 = 48$
$7 \times ? = 56$	9 in $36 = ?$	$? \times 8 = 32$
$4 \times ? = 36$	8 in $64 = ?$	$? \times 9 = 81$
$120 \div 12 = ?$	$99 \div 9 = ?$	$120 \div 10 = ?$
$100 \div 10 = ?$	$66 \div 11 = ?$	$80 \div 8 = ?$
$90 \div 9 = ?$	$44 \div 4 = ?$	$108 \div 9 = ?$
$80 \div 10 = ?$	$54 \div 9 = ?$	$48 \div 4 = ?$
$48 \div 8 = ?$	$84 \div 12 = ?$	$90 \div 10 = ?$
$63 \div 7 = ?$	$50 \div 5 = ?$	$72 \div 12 = ?$
$81 \div 9 = ?$	$54 \div 6 = ?$	$49 \div 7 = ?$
$72 \div 6 = ?$	$24 \div 8 = ?$	$25 \div 5 = ?$
$28 \div 7 = ?$	$24 \div 2 = ?$	$64 \div 8 = ?$
$35 \div 7 = ?$	$40 \div 4 = ?$	$60 \div 5 = ?$
$132 \div 11 = ?$	$22 \div 11 = ?$	$63 \div 7 = ?$
$108 \div 12 = ?$	$32 \div 8 = ?$	$84 \div 7 = ?$

70. Oral.

1. At 12 cents each, how many books can be purchased for 72 cents?

2. How many dozen in 84 oranges?

3. When 8 pounds of sugar cost 40 cents, what is the price of a pound?

4. Among how many boys can 45 cents be divided if each boy receives 9 cents?

5. There are 108 pencils in 108 equal packages. How many pencils in each package?

6. If 6 tons of hay cost 72 dollars, what will 1 ton cost?

7. When milk is 5 cents a quart, how many quarts can be purchased for 40 cents?

8. Paid $55 for 11 tons of coal. What was the price of 1 ton?

9. There are 48 pecks of corn in a bin. There being 8 pecks in a bushel, how many bushels in a bin?

10. There are 8 pints in a gallon. How many gallons in 96 pints of vinegar?

11. A dime is ten cents, and there are 10 dimes in a dollar. How many dollars are there in 80 dimes?

12. A laborer earns 56 dollars in 7 weeks. How much does he earn in 1 week?

13. If 11 boxes of berries sell for 77 cents, what is the price of 1 box?

14. I pay 77 cents for berries at 7 cents a box. How many boxes do I purchase?

15. Bought 9 yards of cloth for 108 cents. How much did I pay for 1 yard?

16. At 12 cents a yard, how many yards of cloth can be bought for 108 cents?

17. A boy wishes to buy a bicycle that costs 60 dollars. In how many weeks can he earn money enough to pay for it at $5 a week?

LONG DIVISION

71. 1. Divide 4932 by 9.

Write the divisor at the left of the dividend with a line between, thus: 9|4932

How many thousands times is 9 contained in 4 thousands? (No thousand times.) 4 thousand 9 hundred make how many hundreds? How many hundreds times is 9 contained in 49 hundreds?

Write 5 hundreds in the hundreds' place in the quotient, thus:

```
  5    Quotient.
9|4932 Dividend.
```

5 hundred times 9 are how many hundreds? Write 45 hundreds under 49 hundreds and subtract, thus:

How many hundreds over?

```
  5    Quotient.
9|4932 Dividend.
 45
  4
```

Bring down three tens from the dividend, thus:

4 hundreds and 3 tens make how many tens? How many tens times is 9 contained in 43 tens?

```
  5    Quotient.
9|4932 Dividend.
 45
 43
```

Write 4 tens in the quotient, thus:

```
 54    Quotient.
9|4932 Dividend.
 45
 43
```

4 tens times 9 are how many tens? Write 36 tens under 43 tens and subtract, thus:

How many tens over?

```
 54    Quotient.
9|4932 Dividend.
 45
 43
 36
  7
```

Bring down 2 units from the dividend, thus:

```
 54    Quotient.
9|4932 Dividend.
 45
 43
 36
 72
```

How many times will 72 units contain 9? Write 8 in units' place in the quotient; multiply 9 by 8 and write the product under 72, thus:

Subtract. How many over? What is the quotient?

```
548    Quotient.
9|4932 Dividend.
 45
 43
 36
 72
 72
```

2. Divide 3553 by 17.

```
 209 Quotient
17|3553 Dividend
  34
  153
  153
```

In this example, 17 is not contained any tens times in 15 tens; so we put 0 in tens' place in the quotient and bring down 3 units.

3. Divide 835,176 by 672.

1242 Quotient + 552 Remainder.
672|835176 Dividend.
672
1631
1344
2877
2688
1896
1344
552 Rem.

In this example, the dividend does not contain the divisor an exact number of times, hence there is a remainder. This remainder may be written over the divisor as part of the quotient thus: $1242\frac{552}{672}$ Quotient.

72. From the foregoing examples, we may make the following

Rules for Long Division

1. Write the dividend at the right of the divisor with a line between.

2. Beginning at the left, find how many figures of the dividend are necessary to contain the divisor. Divide the number represented by these figures by the divisor, and write the quotient above the last figure of the dividend used.

3. Multiply the divisor by the quotient just obtained, and subtract that product from the partial dividend used.

4. Annex to the remainder the next figure of the dividend, and use the number obtained as the next partial dividend. Proceed as before and continue the process until all the figures of the dividend have been used.

5. In dividing Federal money, put the decimal point in the quotient the same number of places from the right-hand side as is the decimal point in the dividend.

73. Find the quotients :

1.	$392 \div 14$	**15.**	$5436 \div 18$	**28.**	$1289 \div 73$
2.	$1124 \div 18$	**16.**	$1286 \div 39$	**29.**	$16{,}428 \div 84$
3.	$3726 \div 24$	**17.**	$2983 \div 42$	**30.**	$12{,}582 \div 58$
4.	$485 \div 17$	**18.**	$1008 \div 25$	**31.**	$1384 \div 75$
5.	$793 \div 15$	**19.**	$9436 \div 47$	**32.**	$6381 \div 86$
6.	$851 \div 27$	**20.**	$8059 \div 36$	**33.**	$12{,}946 \div 24$
7.	$507 \div 16$	**21.**	$1583 \div 46$	**34.**	$9273 \div 91$
8.	$943 \div 22$	**22.**	$4109 \div 51$	**35.**	$7754 \div 78$
9.	$1240 \div 21$	**23.**	$2695 \div 57$	**36.**	$10{,}846 \div 45$
10.	$1296 \div 13$	**24.**	$3874 \div 49$	**37.**	$21{,}431 \div 36$
11.	$1364 \div 19$	**25.**	$9003 \div 25$	**38.**	$27{,}473 \div 97$
12.	$3964 \div 32$	**26.**	$5914 \div 59$	**39.**	$35{,}702 \div 53$
13.	$1289 \div 16$	**27.**	$5630 \div 62$	**40.**	$40{,}060 \div 60$
14.	$5683 \div 37$				

SHORT DIVISION

74. When the divisor is a small number, the work of division may be shortened by omitting all the figures except those of the dividend, divisor, and quotient, and writing the quotient under, instead of over, the dividend. By this method, all the processes are performed, just as in long division, but are not written.

EXAMPLES

1. Divide 347,615 by 5.

$5\overline{)347615}$ Dividend.
69523 Quotient.

2. Divide 20,076 by 12.

$$12\overline{)20076}$$ Dividend.
1673 Quotient.

Every division by numbers not larger than 12 should be performed by short division.

75. Written. Find the quotients:

1.	$2555 \div 5$	**19.**	$30{,}005 \div 5$	**37.**	$31{,}493 \div 6$
2.	$2436 \div 4$	**20.**	$288{,}012 \div 12$	**38.**	$25{,}324 \div 5$
3.	$2845 \div 5$	**21.**	$300{,}010 \div 10$	**39.**	$28{,}764 \div 4$
4.	$4503 \div 3$	**22.**	$99{,}011 \div 11$	**40.**	$36{,}099 \div 9$
5.	$2045 \div 5$	**23.**	$33{,}264 \div 11$	**41.**	$14{,}412 \div 12$
6.	$2835 \div 7$	**24.**	$29{,}280 \div 12$	**42.**	$36{,}930 \div 11$
7.	$4986 \div 9$	**25.**	$36{,}550 \div 10$	**43.**	$24{,}003 \div 6$
8.	$2009 \div 7$	**26.**	$28{,}692 \div 9$	**44.**	$30{,}502 \div 8$
9.	$3504 \div 8$	**27.**	$333{,}333 \div 11$	**45.**	$29{,}333 \div 11$
10.	$\frac{61938}{9}$	**28.**	$35{,}621 \div 7$	**46.**	$675{,}262 \div 5$
11.	$\frac{28343}{7}$	**29.**	$42{,}963 \div 6$	**47.**	$349{,}872 \div 8$
12.	$\frac{59840}{4}$	**30.**	$50{,}725 \div 3$	**48.**	$130{,}052 \div 2$
13.	$\frac{12358}{2}$	**31.**	$82{,}956 \div 10$	**49.**	$168{,}754 \div 9$
14.	$\frac{58345}{5}$	**32.**	$93{,}043 \div 7$	**50.**	$385{,}980 \div 5$
15.	$\frac{11007}{9}$	**33.**	$65{,}407 \div 5$	**51.**	$769{,}520 \div 7$
16.	$\frac{13696}{4}$	**34.**	$39{,}842 \div 9$	**52.**	$387{,}052 \div 10$
17.	$\frac{29365}{7}$	**35.**	$27{,}391 \div 8$	**53.**	$943{,}769 \div 12$
18.	$\frac{98346}{6}$	**36.**	$63{,}598 \div 9$	**54.**	$748{,}131 \div 4$

SPECIAL CASES IN DIVISION

76. Any number, as 34,765, may be analyzed as follows:

34,765 = 34,765 units; or, 3476 tens and 6 over; or, 347 hundreds and 65 over; or, 34 thousands and 765 over; or, 3 ten thousands and 4765 over. Therefore,

$34{,}765 \div 10 = 3476\frac{5}{10}$ or $10)\overline{34765}$ $3476\frac{5}{10}$ Quotient.

$34{,}765 \div 100 = 347\frac{65}{100}$ or $100)\overline{34765}$ $347\frac{65}{100}$ Quotient.

$34{,}765 \div 1000 = 34\frac{765}{1000}$ or $1000)\overline{34765}$ $34\frac{765}{1000}$ Quotient.

$34{,}765 \div 20 = \frac{3476}{2} + \frac{5}{20}$ or $20)\overline{34765}$ $1738\frac{5}{20}$ Quotient.

$34{,}765 \div 500 = \frac{347}{5} + \frac{65}{500}$ or $500)\overline{34765}$ $69\frac{265}{500}$ Quotient.

77. Hence the

Rule for Special Cases in Division

1. To divide by 1 with any number of ciphers annexed, cut off from the right-hand side of the dividend as many figures as there are ciphers in the divisor. The figures cut off will express the remainder, and the figures not cut off will be the quotient.

2. To divide by any number ending in one or more ciphers, cut off from the right-hand side of the dividend as many figures as there are ciphers at the right side of the divisor. Cut off the ciphers from the right of the divisor. Divide the part of the dividend left by the part of the divisor left, and prefix the remainder obtained, if any, to the figures cut off, for a remainder to the entire division.

78. Find the quotients:

1. $1\|00)\underline{369\|00}$		**2.** $1\|00)\underline{283\|25}$		**3.** $1\|000)\underline{36\|954}$	
369		$283\frac{25}{100}$		$36\frac{954}{1000}$	
4. $35{,}800 \div 10$	**10.**	$36{,}900 \div 100$	**16.**	$253{,}000 \div 1000$	
5. $78{,}563 \div 10$	**11.**	$85{,}982 \div 100$	**17.**	$287{,}604 \div 1000$	
6. $50{,}625 \div 10$	**12.**	$36{,}984 \div 100$	**18.**	$129{,}832 \div 1000$	
7. $36{,}072 \div 10$	**13.**	$28{,}305 \div 100$	**19.**	$172{,}001 \div 1000$	
8. $25{,}600 \div 10$	**14.**	$72{,}601 \div 100$	**20.**	$398{,}436 \div 1000$	
9. $35{,}690 \div 10$	**15.**	$35{,}100 \div 100$	**21.**	$128{,}000 \div 1000$	

22. $136{,}948 \div 50$ **23.** $169{,}846 \div 80$

24. $389{,}063 \div 200$ **25.** $8{,}698{,}751 \div 500$

79. **1.** $24 \div ? = 8$. Which terms of division are given? To find what? How do you find it? When the dividend and quotient are given, how can the divisor be found?

2. $? \div 3 = 8$. Which terms are given? To find what? How do you find it? When the divisor and quotient are given, how may the dividend be found? State two ways.

80. These examples show the following

Proof of Division

1. Divide the dividend by the quotient. If the result is the same as the divisor, the work is correct. The remainder, if any, should be the same as in the first division.

2. Multiply the quotient and divisor together. Add the remainder, if there is any, to the product. If the result obtained is the same as the dividend, the division is correct.

81. 1. $9 \times 2 = ?$ Which term is 9? Which term is 2?
$2 \times 9 = ?$ Which term is 9? Which term is 2?
Compare the results in the above examples.

2. $? \times 2 = 18$. Which terms are given? Which term to find? How do you find it? When the multiplier and product are given, how can the multiplicand be found?
$9 \times ? = 18$. When the product and multiplicand are given, how can the multiplier be found?

82. These examples show the following

Proof of Multiplication

1. Multiply the multiplier by the multiplicand. If the work is correct, the product obtained will agree with the first product. Or,
2. Divide the product by the multiplier. If the work is correct, the quotient will be the same as the multiplicand. Or,
3. Divide the product by the multiplicand. If the work is correct, the quotient will be the same as the multiplier.

PROBLEMS IN DIVISION

83. Written.

1. If a steamboat ran 1440 miles in 4 days, what was the average rate for a day?

2. A farmer put 1068 bushels of potatoes in 6 bins. How many bushels were in each bin?

3. A girl divided $4.50 among 10 poor children. How much did she give to each?

4. How many tons of coal at $5 a ton can be bought for $765?

5. If 7 men can build 378 rods of fence in a week, how many rods can one man build in the same time?

6. How many days, at 11 miles a day, will it take a man to walk 902 miles?

7. The dividend is 981, the quotient 9. What is the divisor?

8. How many ploughs at $12 each can be bought for $1008?

9. A man divided 2222 acres of land into 11 parts. How many acres were there in each part?

10. A farmer paid $46.25 for 10 sheep. What did he pay for each?

11. There are 8 quarts in 1 peck. How many pecks are there in 3320 quarts?

12. There are 7 days in 1 week. How many weeks in 364 days?

13. A man willed $19,722 to his 6 children. How much did each receive, if they shared equally?

14. What is the price of 1 yard of broadcloth, if 12 yards cost $50.40?

15. There are 5280 feet in a mile, and 3 feet in 1 yard. How many yards in a mile?

16. At $8 each, how many overcoats can be bought for $1000?

17. How many hours will be required to ride a bicycle 504 miles by riding 18 miles an hour?

18. At $15 each, how many suits of clothes can be bought for $1080?

19. If a family consume 13 gallons of kerosene in a month, in how many months will it consume 468 gallons?

20. At $16 a ton, how many tons of hay can be bought for $1920?

21. If 1 acre of land yields 24 bushels of wheat, how many will be required to yield 2544 bushels?

22. At $27 each, how many stoves can be bought for $648?

23. If a man can walk 30 miles in 1 day, in how many days can he walk 1000 miles?

24. How many weeks will it take to pay a debt of $1225 by paying $25 a week?

REVIEW EXERCISES

84. Oral.

1. A farmer sold 12 barrels of apples at $3 a barrel and bought with the proceeds coal at $4 a ton. How many tons did he buy?

2. A dealer bought 10 barrels of apples at $2 a barrel and sold them so as to gain $10. What was the selling price per barrel?

3. In what time will a boy at $3 a week earn as much as a man earns in 4 weeks at $9 a week?

4. John had $6, James 3 times as much, and Thomas $\frac{1}{4}$ as much as both. How much had Thomas?

5. A man who carried 56 eggs to market found $\frac{1}{8}$ of them broken. How many were not broken?

6. Nell is three years old, and Will 5. Their sister is as old as twice the sum of their ages. How old is their sister?

7. A farmer sold 48 pecks of beans for $48. How much did he receive a bushel, there being 4 pecks in a bushel?

8. Bought a dozen oranges for 36 cents and sold them at 4 cents apiece. What was the entire profit?

9. How many gallons of milk will a family use in 12 days if they use 2 quarts a day, there being 4 quarts in a gallon?

10. Mr. A owed a debt of $96. He made 5 payments of $12 each. How much remained unpaid?

11. Two boys 80 miles apart travel toward each other. The first goes 5 miles an hour, and the second 3 miles an hour. In how many hours will they meet?

12. Two boys, travelling at the rate of 5 miles and 3 miles an hour respectively, go in the same direction. How far apart are they at the end of 6 hours?

13. If I can buy 7 oranges for 14 cents, how many can I buy for 24 cents?

14. By selling 12 barrels of flour at $5 a barrel, a grocer makes a profit of $12. What was the cost of the flour per barrel?

REVIEW PROBLEMS

85. Written.

1. If the dividend is 1821, the quotient 32, and the remainder 29, what is the divisor?

2. The product of three numbers is 1260, and two of them are 12 and 7. What is the third?

3. A grocer buys 88 gallons of molasses at $.56 a gallon. For how much per gallon must he sell it in order to gain $12.32?

4. Two railway trains start at the same time from opposite points, 1216 miles apart, and travel toward each other, one going 35 miles an hour, and the other 41 miles an hour. How many hours before they will meet?

5. Two steamboats are 144 miles apart and going in the same direction, one at the rate of 21 miles, and the other 15 miles, an hour. In how many hours will the former overtake the latter?

6. A merchant bought 8 pieces of cloth, each containing 52 yards, at 12 cents a yard, and 325 pounds of cotton batting at 9 cents a pound. How much less than $100 did it cost him?

7. If 64 pounds of butter costs $15.36, what will 320 pounds cost at the same price?

8. A grocer bought 123 gallons of molasses at 42 cents a gallon and sold it at 60 cents a gallon. What was his gain?

9. A man bought 4 cows at one time for $168, 3 at another for $153, and 6 at another for $225. What was the average price paid?

10. Divide the product of 27 and 32 by their sum.

11. If 18 men can build a wall in 120 days, in what time can 24 men build it?

12. A man, dying, left property amounting to $3500. After paying $572 for funeral and other expenses, the remainder was divided equally among 12 heirs. What did each receive?

13. Two vessels start from the same point and sail in opposite directions, one at the rate of 18 miles, and the other 23 miles, an hour. How far apart will they be in 16 hours?

14. I bought 124 bushels of corn of A, 216 of B, and 96 of C, paying $.63 a bushel in each case. What will be my profit if I sell the whole at $.72 a bushel?

15. A man starts on a journey of 724 miles. When he has travelled 12 hours at the rate of 32 miles an hour, how far is he from his journey's end?

16. A man bought 6 sacks of flour at \$1.25 a sack, 24 pounds of sugar at 6 cents a pound, and 2 pounds of coffee at 35 cents a pound, and paid for it in butter at 23 cents a pound. How many pounds were there?

17. If I sell 58 cows at \$38 a head, and 200 sheep at \$4.30 a head, and invest the proceeds in 4 village lots, what is the average price of each lot?

18. I bought a city lot for \$1750, built a house upon it at a cost of \$3275, and afterward sold the place for \$6000. What was my gain?

19. A boy has \$9.62 in his toy bank. How many times can he take away 25 cents from it, and have 12 cents left?

20. A man bought a house and lot for \$3200. He paid \$1275 down, and agreed to pay the remainder in equal monthly payments of \$35 each. How many months will it take to pay in full?

21. The product of four numbers is 2880, and three of them are 5, 6, and 8. What is the fourth?

22. A teacher who receives a monthly salary of \$125 for 10 months in a year will require how many years to pay for 3 village lots at \$645 each, if his yearly expenses are \$863?

23. A man having \$783.58 in bank, drew out \$132.75 at one time, \$175.50 at another, \$216 at another. How much then remained in the bank?

24. How many tons of coal at \$5 a ton will pay for 15 tons of hay at \$11 a ton?

25. If a pupil in school uses 6 sheets of writing paper each day, how many tablets containing 75 sheets will he require for the school year of 40 weeks of 5 days each, if he attends school every day, making no allowance for holidays?

26. I paid $365 for a horse and carriage, paying $1 more for the horse than for the carriage. What did I pay for each?

27. A farmer paid $1125 for cows, horses, and farming tools, and 8 times as much for a farm of 125 acres. What was the price per acre?

28. John, Edward, and Henry have together $12.80. John has $2.26 more than Henry, and Edward has $1.75 more than Henry. How much money has each?

29. A man bought 84 acres of land at $65 an acre and sold it all for $6552. What was his gain per acre?

30. A merchant bought 172 yards of cashmere at $.45 a yard, 504 yards of calico at $.05, 252 yards of flannel at $.37, and paid for it in 3 equal payments. What was the amount of each payment?

31. A drover bought 27 cows for $1026. At what price per head must he sell them to make a profit of $12 on each?

32. There are 24 sheets of paper in a quire. How many sheets in 2 reams, there being 20 quires in a ream?

33. A man bought 1000 sheep and sold at one time 216, at another 327, and at another 198. How many did he then have left?

34. There are 640 acres in 1 square mile. Into how many farms of 160 acres each can 10 square miles of land be divided?

35. A merchant bought dress goods at $1.26 a yard and sold it at $2 a yard. What was his profit on 260 yards?

36. I sold a farm for $1265 more than I paid for it, but $325 less than my asking price. What would have been my profit if I had sold at my asking price?

37. How many more acres of land can I buy with $6048 at $56 an acre than at $72 an acre?

38. A grocer bought 123 pounds of cheese at 10 cents a pound and sold it for 13 cents a pound. What was his profit?

39. A lady bought 12 yards of dress goods at $1.75 a yard, 8 yards of silesia at $.25 a yard, 2 pairs of gloves at $1.45 a pair, 6 handkerchiefs at $.25 apiece, and 3 yards of table linen at $.95 a yard. She paid $18.75 and left the balance on account. What did she still owe?

40. How many pounds of butter at 26 cents a pound must be given for 3 barrels of flour at $4.20 a barrel?

41. At what rate per hour must a railway train run to overtake, in 12 hours, another train 168 miles ahead running at the rate of 32 miles an hour?

42. How many more revolutions will a wheel 10 feet in circumference make in going 5280 feet, or a mile, than one 20 feet in circumference?

43. How much more will 8 horses at $165 each cost than 23 cows at $37 a head?

44. From a bin containing 324 bushels of wheat, 147 bushels were sold at $.85, and the remainder at $.90 a bushel. What did the wheat bring?

45. A man buys 32 cows for $1120 and sells them at a gain of $192. What does he receive a head for them?

46. How many hours will it take a horse, travelling 8 miles an hour, to go as far as a train of cars can go in 4 hours running at the rate of 35 miles an hour?

FACTORING

86. A number that will divide another number without a remainder is an **Exact Divisor** of that number.

87. An exact divisor of a number is a **Factor** of that number. Thus, 5 is a factor of 15. Why? What other factor has 15? If a number has any factors, what is the least number of factors it may have? Which terms in division are factors? They are factors of what? Which terms in multiplication are factors? They are factors of what? When one factor of a number is known, how can the other factor be found?

88. A number that has no factors but itself and one is a **Prime Number.** Thus, 1, 3, 5, 7, 11, 13, 17, and 19 are prime numbers.

89. A number that has other factors than itself and 1 is a **Composite Number.** Thus, 4, 9, 12, and 21 are composite numbers.

90. A factor that is a prime number is a **Prime Factor.** Thus, 2, 3, and 5 are prime factors of 30.

91. A number that has two for a factor is an **Even Number.** Name all the even numbers from 2 to 30.

92. A number of which 2 is not a factor is an **Odd Number.** Name all the odd numbers from 1 to 29.

93.

Rule for finding whether a Number is Prime or not

1. Divide the given number by 2.
2. If 2 gives a remainder, divide by 3.
3. Continue this process, using each prime number in order as a divisor, until an exact divisor is found, *or until the divisor exceeds the quotient.* If no exact divisor is found until the divisor used exceeds the quotient, the number is prime. Otherwise it is composite.

94. Find whether these numbers are prime or composite:

1. 143	**5.** 211	**9.** 121	**13.** 231	**17.** 437
2. 123	**6.** 221	**10.** 97	**14.** 161	**18.** 401
3. 324	**7.** 119	**11.** 213	**15.** 87	**19.** 593
4. 163	**8.** 208	**12.** 215	**16.** 78	**20.** 395

95. Finding the factors of numbers is called **Factoring.**

NOTE. — Since 1 is a factor of every number, it is not generally mentioned among the factors of a number.

1. Find the prime factors of 1320.

$$\begin{array}{r|l} 2 & 1320 \\ 2 & 660 \\ 2 & 330 \\ 3 & 165 \\ 5 & 55 \\ & 11 \end{array}$$

By what kind of numbers must we divide? Why?
Which divisors do we use first?
What beside the divisors is a prime factor?

2, 2, 2, 3, 5, 11. *Ans.*

96.

Rule for finding the Prime Factors of a Number

1. Divide the given number by its smallest prime factor. Divide the quotient by the same factor, if possible, and repeat the process as many times as that divisor can be used.

2. Divide the last quotient obtained by the next larger prime factor, and repeat if possible, as with the first prime factor.

3. Continue in this way until the quotient is a prime number. All the divisors and the last quotient are the prime factors required.

97. Find the prime factors of :

1. 30	**5.** 110	**9.** 189	**13.** 414
2. 120	**6.** 105	**10.** 665	**14.** 3381
3. 42	**7.** 462	**11.** 429	**15.** 667
4. 66	**8.** 45	**12.** 425	

CANCELLATION

98. **1.** Divide 210 by 30.

$\frac{210,\ \text{Dividend}}{30,\ \text{Divisor}} = 7,\ \text{Quotient.}$

$\frac{210 \div 2}{30 \div 2} = \frac{105}{15} = 7,\ \text{Quotient.}$ { What is done to the dividend? To the divisor? How does it affect the quotient?

$\frac{105 \div 3}{15 \div 3} = \frac{35}{5} = 7,\ \text{Quotient.}$ { What is done to the dividend? To the divisor? How does it affect the quotient?

$\frac{35 \div 5}{5 \div 5} = \frac{7}{1} = 7,\ \text{Quotient.}$ { What is done to the dividend? To the divisor? How does it affect the quotient?

Dividing both dividend and divisor by the same number affects the quotient how?

$$\frac{210}{30} = \frac{2 \times 3 \times 5 \times 7}{2 \times 3 \times 5} = \frac{\cancel{2} \times \cancel{3} \times \cancel{5} \times 7}{\cancel{2} \times \cancel{3} \times \cancel{5}} = 7, \text{ Quotient.}$$

This example might be expressed thus:

$$\begin{array}{l} 7 \\ \cancel{35} \\ \cancel{105} \\ \cancel{210} \\ \cancel{30} \\ \cancel{15} \\ \cancel{5} \\ 1 \end{array} = 7, \text{ Quotient.}$$

99. Taking out the same factor from both dividend and divisor is **Cancellation.**

100. Find the answers to the following questions by means of cancellation:

1. Divide $36 \times 27 \times 49 \times 38 \times 50$ by $70 \times 18 \times 15$.

2. $(28 \times 38 \times 48) \div (14 \times 19 \times 24 \times 2 \times 2) = ?$

3. $(26 \times 5 \times 54) \div (13 \times 5 \times 6) = ?$

4. What is the quotient of $36 \times 48 \times 16$ divided by $27 \times 24 \times 8$?

5. Divide $5 \times 45 \times 7 \times 20$ by $49 \times 5 \times 4 \times 9$.

6. Divide $5 \times 51 \times 7 \times 9 \times 4$ by $17 \times 20 \times 12 \times 7 \times 2$.

7. Divide $25 \times 2 \times 72 \times 14$ by $6 \times 9 \times 120$.

8. How many bushels of potatoes at 50 cents a bushel must be given in exchange for 15 pounds of tea at 40 cents a pound?

9. If 60 yards of cloth cost \$120, how many yards can be bought for \$40?

10. 15 oranges cost 45 cents How much will 7 oranges cost?

11. A dairyman sells 100 quarts of milk daily at 5 cents a quart. How many bushels of corn at 45 cents a bushel can he buy with 10 days' milk receipts?

12. A farmer sold a grocer 45 bushels of apples at 50 cents a bushel, taking his pay in flour at 90 cents a sack. How many sacks did he receive?

GREATEST COMMON DIVISOR

101. A number that will exactly divide two or more numbers is a **Common Divisor** of those numbers. It is also a **Common Factor.** Thus, 2 is a common divisor of 6 and 4; 5 cents is a common divisor of 10 cents, 25 cents, and 15 cents.

102. The largest number that will exactly divide two or more numbers is the **Greatest Common Divisor** of those numbers. It is also the **Greatest Common Factor.** Thus, 6 is the greatest common divisor of 12, 18, and 24; 15 is the greatest common divisor of 45 and 60.

103. Numbers that have no common factor are **Prime to Each Other.** Thus, 22 and 35 are prime to each other.

104. Find the greatest common divisor of 24, 36, and 72.

$24 = 2 \times 2 \times 2 \times 3$
$36 = 2 \times 2 \times \quad 3 \times 3$
$72 = 2 \times 2 \times 2 \times 3 \times 3$

Name the common factors of 24, 36, and 72. Since 2, 2, and 3 are all the common factors of 24, 36, and 72, what will you do with 2, 2, and 3 to find the greatest common factor? $2 \times 2 \times 3 =$ what? What is the greatest common factor? For convenience, we may find the common factors thus:

$$\begin{array}{r|l} 2 & 24,\ 36,\ 72 \\ 2 & 12,\ 18,\ 36 \\ 3 & 6,\ 9,\ 18 \\ \hline & 2,\ 3,\ 6 \end{array}$$

$2 \times 2 \times 3 = 12$, greatest common divisor. *Ans.*

105. From this example we may make the following

Rule for finding the Greatest Common Divisor

1. Write the given numbers in a horizontal line.
2. Divide them by all their common factors in succession, beginning with the smallest.
3. Multiply the common factors together.

106. Find the greatest common divisor:

1.	84, 132	**6.**	40, 60, 80	**11.**	45, 60, 90
2.	63, 42	**7.**	64, 144, 560	**12.**	36, 72, 81
3.	90, 105	**8.**	36, 48, 24	**13.**	44, 121, 132
4.	112, 168	**9.**	40, 56, 72	**14.**	63, 126, 189
5.	132, 156	**10.**	18, 54, 32	**15.**	36, 81, 135

LEAST COMMON MULTIPLE

107. A number that exactly contains another number is a **Multiple** of that number. Thus, 30 is a multiple of 10.

108. A number that exactly contains two or more numbers is a **Common Multiple** of those numbers. Thus, 60 is a common multiple of 15, 6, and 10.

109. The smallest number that exactly contains two or more numbers is the **Least Common Multiple** of those numbers. Thus, 30 is the least common multiple of 15, 6, and 10.

1. Find the least common multiple of 60, 90, 50, and 150.

$60 = 2 \times 2 \times 3 \times 5$
$90 = 2 \times 3 \times 3 \times 5$
$50 = 2 \times 5 \times 5$
$150 = 2 \times 3 \times 5 \times 5$

What kind of factors have we found? A number to contain 60 must contain what factors? To contain 90? To contain 50? To contain 150?

To contain 60, 90, 50, and 150, a number must contain how many factor 2's? How many factor 3's? How many factor 5's? What other factors? What is the smallest number that contains $2 \times 2 \times 3 \times 3 \times 5 \times 5$? What, then, is the least common multiple of 90, 60, 50, and 150?

The necessary factors may be conveniently found thus:

2	90	50	60	150
3	45	25	30	75
5	15	25	10	25
5	3	5	2	5
	3	1	2	1

$2 \times 3 \times 5 \times 5 \times 3 \times 2 = 900$, least common multiple. *Ans.*

110. From this example we may make the following

Rule for finding the Least Common Multiple

1. Write the given numbers in a horizontal line.
2. Divide them by any prime number that will divide two or more of them. It is best to begin with the smallest.
3. If any number will not exactly contain the divisor, bring that number down into the line with the quotients.
4. Continue this process until the quotients obtained are prime to each other.
5. Multiply together all the divisors and the last line of quotients.

111. Find the least common multiple:

1. 18, 27, 30	5. 36, 40, 48	9. 24, 42, 54, 360
2. 9, 12, 18	6. 18, 24, 36	10. 25, 20, 35, 40
3. 16, 48, 60	7. 15, 30, 21, 28	11. 14, 21, 35, 45
4. 21, 27, 36	8. 15, 60, 140, 210	12. 24, 48, 96, 192

INDICATED OPERATIONS

112. The **Parenthesis** ( ) indicates that all the numbers included therein are to be subjected to the same operation.

Thus, $(17 - 6) \times 2$ indicates that the difference between 6 and 17 is to be multiplied by 2. $17 - 6 = 11$. $11 \times 2 = 22$. Without the parenthesis, this would be $17 - 6 \times 2$, and indicates that 2×6 is to be taken from 17. $17 - 11 = 6$.

$(4 + 8) \div 4$ indicates that the sum of 8 and 4 is to be divided by 4. The result is 3. If the parenthesis is omitted, we have $4 + 8 \div 4$, which indicates that $8 \div 4$, or 2, is to be added to 4. $4 + 4 = 8$.

$[(6 + 5) \times 4 + 6] \div 10$ indicates that $6 + 5$ is to be multiplied by 4, 6 added to the product, and the sum divided by 10. $6 + 5 = 11$. $11 \times 4 = 44$. $44 + 6 = 50$. $50 \div 10 = 5$.

113. **Brackets** [ ] and the **Vinculum** ——— have the same uses as the parenthesis.

$[3 + 4] \times 3$, $\overline{3 + 4} \times 3$, mean the same as $(3 + 4) \times 3$.

When the parenthesis or vinculum is included within the brackets, the operations indicated within the parenthesis or under the vinculum should be performed first.

114. When the parenthesis, vinculum, or brackets are not used, or after they have been removed, operations indicated by $\times$ or $\div$ must be performed first, then the operations indicated by $+$ and $-$. Thus, $12 \div 4 \times 2 + 36 \div 4 - 2 \times 4 = ?$

SOLUTION. — $12 \div 4 \times 2 = 6$. $36 \div 4 = 9$. $2 \times 4 = 8$. The statement now reads, $6 + 9 - 8$. The result is 7.

115.

1. $4 + 3 \times 2 = ?$
2. $(4 + 3) \times 2 = ?$
3. $4 \times 3 + 2 = ?$
4. $4 \times (3 + 2) = ?$
5. $8 + 4 \div 2 = ?$
6. $(8 + 4) \div 2 = ?$

Find the value of :

7. $15 + 3 \times 6 + 10 \div 5.$
8. $(6 + 4) \times (3 + 2) - (8 \times 5).$
9. $18 \div 3 + 2 + 8 \times 2 + 14 - 6.$
10. $2 + 12 \div 4 - (10 + 16 \div 4) \div 7.$
11. $\overline{11 + 4} \div 3 + 6 \times 4.$
12. $3 + 4 \times 6 \div (15 - 18 \div 3).$
13. $164 + 16 - 250 \div 10 + 16 \times 3.$
14. $17 + 3 \times 4 \times 6 + 3 \div 3 + 3.$
15. $[39 + 8 \div 2 + 7] \times 6.$
16. $[6 + 15 \times 3 - \overline{6 + 16 \div 8 + 4}] \div 3 + 5.$
17. $\dfrac{7 + 8}{5} + \dfrac{10 - 2.}{2}$
18. $7 \times \overline{3 + 4} - 9.$
19. $\left[\left(\overline{6 + 7} \times 5 - \dfrac{12 + 8}{4}\right) - 20\right] \div 8.$
20. Indicate the addition of 36, 15, 16, 38.
21. Indicate in two ways the division of 16 by 4. Of $5 + 17$ divided by 11.

Indicate the operations of the following problems :

22. A lady found 3 roses on one bush, 5 on another, and 7 on another. How many roses did she find ?
23. James found 2 eggs in a hen's nest, 5 in another, and 6 in another. How many eggs did he find in all ?

24. A man spent \$10.16 on Monday and \$4.40 on Tuesday. How much more did he spend on Monday than on Tuesday?

25. A boy had 36 marbles. He lost 6 one day and 11 the next. How many had he left?

$$36 - (6 + 11) = ?$$

26. If you have 60 cents and spend 18 cents for ribbon and 12 cents for thread, how much have you left?

27. James had 18 marbles and found 5 more. He afterward lost 8 marbles. How many were left?

28. A boy deposited \$1 in the savings bank on the 1st of the month, 75¢ on the 10th, and \$2.50 on the 25th. On the 30th he drew out \$2.75. How much was left in the bank?

29. John had \$5.18 in the bank. He drew out \$1.50 on Monday and \$.75 on Tuesday. How much had he left?

30. 4 increased by the product of 3 and 2.

31. The sum of 4 and 3, multiplied by 2.

32. The product of 4 and 3, increased by 2.

33. 4 multiplied by the sum of 3 and 2.

34. 84 less the product of 5 and 6.

35. The difference between 72 and 42, multiplied by 4.

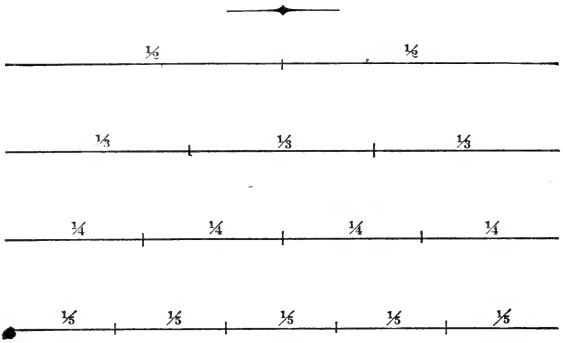
36. 144 subtracted from the product of 18 and 18.

37. 6 multiplied by the difference between 11 and 6.

38. Henry has 58¢, and John 63¢. Charles has 4 times as much as both. How much has Charles?

39. Lucy and Mary went shopping. Lucy bought lace for 55¢, and Mary 5 yards of ribbon at 15¢ a yard. What was the amount of both purchases?

FRACTIONS



116. Draw a line. Divide it into 2 equal parts. What is one of these parts? How many halves in the line?

Draw another line. Divide it into 3 equal parts. What is one part? How many thirds in the line?

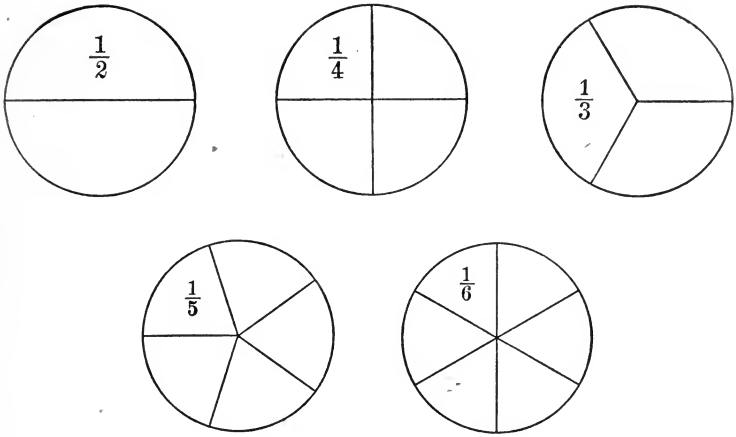
Draw another line. Divide it into 4 equal parts. What is one part? How many fourths in the line?

Draw another line. Divide it into 5 equal parts. One of these parts is what? Two of them are what? Three of them? Four of them? All of them?

Which is larger, $\frac{1}{2}$ or $\frac{1}{5}$? $\frac{1}{3}$ or $\frac{1}{4}$?

117. Make a square. Draw a line through the middle of it from corner to corner. Into how many parts have you divided the square? What is one part?

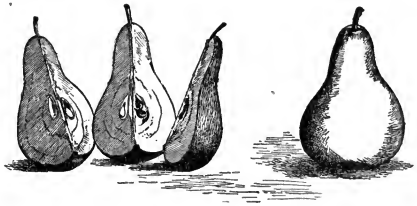
Connect the other corners by a line. Into how many parts is the square now divided? What is one of the parts called? What are two of them called? Three of them?



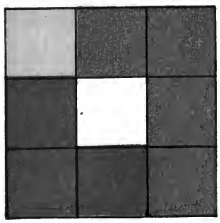
118. Which piece of this pie would you rather have, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$, or $\frac{1}{5}$? Why?

One piece of the last pie is what part of it? two pieces? 3 pieces? 5 pieces?

Write the number that represents 1 part of this pear. Two parts. Write the number that represents the whole pear.



How many small squares are there in the large square? One of the small squares is what part of the large square? What part of the large square is yellow? White? Yellow and white? How many ninths of the large square are red? What part of the large square is blue?



How many oblongs are here?

How many circles?

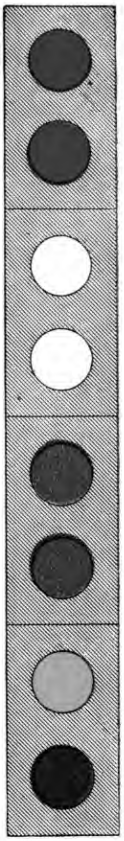
One oblong is what part of all the oblongs.

What part of all the circles are in one oblong?

$\frac{1}{4}$ of all the circles are how many circles? $\frac{1}{4}$ of $8 = ?$ $\frac{8}{4} = ?$ $8 \div 4 = ?$

What part of all the circles are red? White? Yellow? Green? Red and white?

$\frac{1}{8}$ of all the circles are how many circles? $\frac{8}{8}$ = how many? $\frac{8}{2}$ = how many?



119. A **Fraction** is one or more of the equal parts of any thing. Thus, $\frac{1}{7}$ of an inch; $\frac{2}{11}$; $\frac{1}{5}$.

120. A fraction is also an expression of division. Thus, $\frac{1}{4}$ means $1 \div 4$, or 1 whole thing divided into 4 equal parts. $\frac{24}{3}$ means $24 \div 3$, or 24 divided into 3 equal parts.

121. The number below the line in a fraction is the **Denominator.** It tells the number of parts into which the whole is divided. Thus, 9 is the denominator of $\frac{7}{9}$.

122. The number above the line in a fraction is the **Numerator.** It tells the number of parts used. Thus, 3 is the numerator of $\frac{3}{8}$.

123. The numerator and denominator are the **Terms of a Fraction.** What are the terms of $\frac{1}{2}$? of $\frac{3}{7}$? of $\frac{5}{9}$? of $\frac{23}{31}$?

124. The quotient of the numerator divided by the denominator is the **Value of the Fraction.** What is the value of $\frac{2}{2}$? of $\frac{9}{3}$? of $\frac{16}{2}$? of $\frac{25}{5}$? of $\frac{3}{8}$?

125. Read the following fractions :

1. $\frac{4}{5}$	**5.** $\frac{2}{5}$	**9.** $\frac{5}{10}$	**13.** $\frac{15}{21}$
2. $\frac{2}{3}$	**6.** $\frac{3}{7}$	**10.** $\frac{8}{11}$	**14.** $\frac{14}{32}$
3. $\frac{5}{8}$	**7.** $\frac{7}{8}$	**11.** $\frac{5}{12}$	**15.** $\frac{12}{33}$
4. $\frac{7}{9}$	**8.** $\frac{8}{9}$	**12.** $\frac{7}{20}$	**16.** $\frac{15}{50}$

126. Write in figures :

17. Four-fifths.

18. Seven-ninths.

19. Five-eighths.

20. Six-tenths.

21. Four-elevenths.

22. Six-thirteenths.

23. Four twenty-firsts.

24. Sixteen twenty-seconds.

25. Fifteen twenty-fifths.

26. Sixteen forty-fourths.

27. Eight thirty-thirds.

28. Six fifty-fourths.

29. Seventeen sixty-ninths.

30. Seven ninety-eighths.

31. Twenty-four eightieths.

32. Seven fiftieths.

33. Sixty-five seventieths.

34. 10 one-hundred-fifths.

35. 12 two-hundred-eighths.

REDUCTION OF FRACTIONS

Reduction of Fractions to Lowest Terms

127. **Reduction** is changing the form of numbers without changing their value. Thus, $\frac{2}{2} = 1$; 10 cents = 1 dime.

128. Fractions are in their **Lowest Terms** when the numerator and denominator have no common divisor. Thus, $\frac{2}{5}$ is in its lowest terms. $\frac{4}{12}$ is not in its lowest terms.

FIRST ILLUSTRATION

How many squares are there in the oblong?

One square is what part of the oblong?

How many squares are red?

How many ninths are red?

How many colors are there in the oblong?

Into how many parts do the colors divide the oblong? How do these parts compare? One of these parts is what fraction of the oblong? What fraction of the oblong is red? How many ninths are red? How does $\frac{3}{9}$ compare with $\frac{1}{3}$? Write $\frac{3}{9} = \frac{1}{3}$.

What could you do to 3 to get 1?

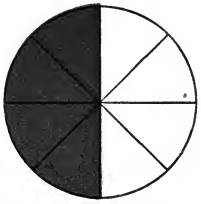
What could you do to 9 to get 3?

What would you do to $\frac{3}{9}$ to get $\frac{1}{3}$?

Dividing both terms of a fraction by the same number affects the value of the fraction how?

The terms of $\frac{1}{3}$ have what common divisor? Then $\frac{3}{9}$ has been reduced to what? (See definition, Art. 128.)

SECOND ILLUSTRATION

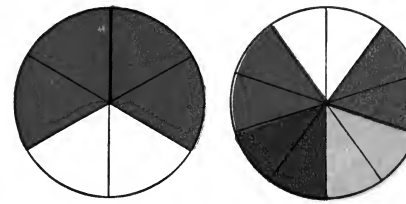


Into how many equal parts is this circle divided by the black lines? What is one of the 8 equal parts called? What part of the circle is green? How many eighths are in the green half? $\frac{4}{8}$ of the circle compares how with $\frac{1}{2}$? What must you do with 4 and 8 in the fraction $\frac{4}{8}$ to make it $\frac{1}{2}$? The terms of $\frac{1}{2}$ have what common divisor?

Then $\frac{4}{8}$ has been reduced to what?

129. Show by these circles that $\frac{2}{6}=\frac{1}{3}$. That $\frac{4}{6}=\frac{2}{3}$. That $\frac{2}{10}=\frac{1}{5}$. That $\frac{4}{10}=\frac{2}{5}$. That $\frac{6}{10}=\frac{3}{5}$. That $\frac{5}{10}=\frac{1}{2}$. That $\frac{8}{10}=\frac{4}{5}$.

How are these fractions reduced to lowest terms?



130.

Rule for Reduction of Fractions to Lowest Terms

Divide both numerator and denominator by common divisors until they become prime to each other.

The numerator of a fraction is which term of division? The denominator? The value of the fraction?

Dividing both dividend and divisor by the same number affects the quotient how? (See Cancellation, article No. 103.)

Dividing both numerator and denominator of a fraction by the same number affects the value of the fraction how?

Change the following to their lowest terms:

131. Mental.

1. $\frac{3}{6}$	**4.** $\frac{4}{8}$	**7.** $\frac{20}{30}$	**10.** $\frac{36}{72}$	**13.** $\frac{27}{54}$
2. $\frac{4}{6}$	**5.** $\frac{10}{15}$	**8.** $\frac{56}{84}$	**11.** $\frac{24}{40}$	**14.** $\frac{28}{35}$
3. $\frac{10}{12}$	**6.** $\frac{15}{18}$	**9.** $\frac{42}{49}$	**12.** $\frac{18}{36}$	**15.** $\frac{72}{100}$

132. Written.

1. $\frac{36}{54}$	**4.** $\frac{45}{95}$	**7.** $\frac{60}{360}$	**10.** $\frac{75}{225}$
2. $\frac{72}{108}$	**5.** $\frac{38}{86}$	**8.** $\frac{72}{128}$	**11.** $\frac{80}{236}$
3. $\frac{27}{81}$	**6.** $\frac{55}{99}$	**9.** $\frac{214}{316}$	**12.** $\frac{124}{224}$

13. $\frac{470}{2350}$ **17.** $\frac{130}{251}$ **21.** $\frac{384}{1152}$ **25.** $\frac{83}{581}$

14. $\frac{150}{6000}$ **18.** $\frac{77}{165}$ **22.** $\frac{647}{1294}$ **26.** $\frac{168}{612}$

15. $\frac{112}{1888}$ **19.** $\frac{96}{544}$ **23.** $\frac{441}{951}$ **27.** $\frac{105}{525}$

16. $\frac{121}{154}$ **20.** $\frac{114}{285}$ **24.** $\frac{97}{873}$ **28.** $\frac{121}{143}$

29. Express in lowest terms $230 \div 345$.

30. Express in lowest terms 98 divided by 392.

31. Express in lowest terms $437 \div 2485$.

32. Express in lowest terms the quotient of 288 divided by 504.

33. What are the lowest terms of $\frac{138}{153}$?

Reduction of Integers and Mixed Numbers to Fractions

133. A number that is composed of whole units only is an **Integer.** Thus, 5, 4, 1, 13, and 2000 are integers.

134. A number that is composed of an integer and a fraction is a **Mixed Number.** Thus $8\frac{2}{3}$, $9\frac{1}{10}$, and $3\frac{5}{8}$ are mixed numbers.

How many fourths in 1 circle? In 2 circles? In 3 circles? In 4 circles? How do you find it?

How many fourths in $4\frac{3}{4}$ circles? In $2\frac{3}{4}$ circles? In $3\frac{1}{4}$ circles? How do you find it?

How many eighths in 1 circle? In 3 circles? In 2 circles? In $4\frac{6}{8}$ circles? In $2\frac{3}{8}$ circles? How do you find it?

How do you reduce an integer or a mixed number to a fraction?

135.

Rule for Reduction of Integers and Mixed Numbers to Fractions

1. Multiply the integer by the denominator of the required fraction.
2. Add to the product the numerator of the given fraction, if there is one.
3. Use this result as the numerator of the required fraction.
4. For a denominator, take the denominator of the given fraction or fraction required.

Reduce the following to fractions:

136. Mental.

1. $1\frac{1}{3}$	**5.** $3\frac{4}{5}$	**9.** $3\frac{4}{7}$	**13.** $8\frac{1}{9}$
2. $4\frac{1}{2}$	**6.** $2\frac{3}{4}$	**10.** $4\frac{7}{9}$	**14.** $7\frac{5}{11}$
3. $3\frac{2}{3}$	**7.** $4\frac{3}{5}$	**11.** $5\frac{2}{5}$	**15.** $8\frac{9}{10}$
4. $5\frac{1}{4}$	**8.** $2\frac{5}{6}$	**12.** $6\frac{3}{7}$	**16.** $9\frac{1}{6}$

137. Written.

17. $25\frac{1}{3}$	**24.** $37\frac{4}{13}$	**31.** $270\frac{23}{35}$	**38.** $491\frac{7}{45}$
18. $9\frac{3}{14}$	**25.** $49\frac{7}{15}$	**32.** $19\frac{7}{12}$	**39.** $35\frac{1}{31}$
19. $17\frac{1}{9}$	**26.** $25\frac{1}{30}$	**33.** $29\frac{4}{11}$	**40.** $191\frac{5}{12}$
20. $25\frac{6}{7}$	**27.** $59\frac{3}{14}$	**34.** $149\frac{6}{7}$	**41.** $203\frac{8}{19}$
21. $15\frac{5}{11}$	**28.** $67\frac{5}{16}$	**35.** $128\frac{4}{5}$	**42.** $98\frac{11}{13}$
22. $23\frac{2}{15}$	**29.** $89\frac{20}{29}$	**36.** $137\frac{7}{18}$	**43.** $87\frac{17}{19}$
23. $40\frac{7}{9}$	**30.** $131\frac{7}{9}$	**37.** $238\frac{11}{48}$	**44.** $138\frac{20}{129}$

Reduction of Fractions to Integers or Mixed Numbers

138. A fraction whose numerator is smaller than its denominator is a **Proper Fraction.** Thus, $\frac{2}{3}$, $\frac{5}{9}$, and $\frac{19}{20}$ are proper fractions.

139. A fraction whose numerator equals or exceeds its denominator is an **Improper Fraction.** Thus, $\frac{8}{8}$, $\frac{12}{3}$, $\frac{16}{5}$, and $\frac{31}{7}$ are improper fractions.

140. A boy has two half dollars. That is the same as how many whole dollars? Six half dollars equal how many whole dollars? How do you find it?

Eleven half dollars make how many dollars and how many halves over? How do you find it? Write it.

How many quarters make a dollar?

How many dollars would 8 quarters make? 40 quarters?

Fifteen quarters make how many dollars and how many quarters over? Write it. How do you find it?

$\frac{2}{2}$ = how many whole ones? $\frac{8}{2}$? $\frac{16}{2}$? $\frac{9}{2}$?

$\frac{4}{4}$ = how many wholes? $\frac{8}{4}$? $\frac{13}{4}$? $\frac{14}{4}$?

A fraction is an expression of what operation?

How do you find the value of a fraction?

Rule for Reduction of a Fraction to an Integer or a Mixed Number

Divide the numerator by the denominator.

141. Mental. Find the values of:

1. $\frac{4}{2}$	**5.** $\frac{10}{4}$	**9.** $\frac{25}{6}$	**13.** $\frac{73}{7}$	**17.** $\frac{245}{5}$	**21.** $\frac{42}{13}$
2. $\frac{5}{4}$	**6.** $\frac{7}{3}$	**10.** $\frac{39}{7}$	**14.** $\frac{80}{11}$	**18.** $\frac{77}{12}$	**22.** $\frac{63}{15}$
3. $\frac{6}{3}$	**7.** $\frac{13}{3}$	**11.** $\frac{40}{8}$	**15.** $\frac{91}{9}$	**19.** $\frac{46}{11}$	**23.** $\frac{56}{15}$
4. $\frac{7}{2}$	**8.** $\frac{17}{4}$	**12.** $\frac{51}{3}$	**16.** $\frac{124}{2}$	**20.** $\frac{74}{12}$	**24.** $\frac{111}{11}$

142. Written. Reduce to whole or mixed numbers:

1. $\frac{191}{4}$	**3.** $\frac{292}{14}$	**5.** $\frac{967}{32}$	**7.** $\frac{1216}{20}$	**9.** $\frac{930}{57}$
2. $\frac{376}{25}$	**4.** $\frac{817}{27}$	**6.** $\frac{785}{37}$	**8.** $\frac{989}{45}$	**10.** $\frac{3900}{29}$
11. $\frac{943}{18}$	**14.** $\frac{9603}{39}$	**17.** $\frac{893}{46}$	**20.** $\frac{4973}{67}$	
12. $\frac{3467}{14}$	**15.** $\frac{979}{37}$	**18.** $\frac{912}{89}$	**21.** $\frac{7234}{79}$	
13. $\frac{879}{37}$	**16.** $\frac{348}{18}$	**19.** $\frac{3522}{171}$	**22.** $\frac{28501}{104}$	

To Least Common Denominator

143. Fractions whose denominators are alike have a **Common Denominator.** Thus, $\frac{3}{60}$, $\frac{18}{60}$, and $\frac{27}{60}$ have a common denominator.

144. Fractions having the smallest possible common denominator are said to have their **Least Common Denominator.** Thus, $\frac{1}{20}$, $\frac{6}{20}$, $\frac{9}{20}$, and $\frac{47}{20}$ have their least common denominator.

145. Multiplying both dividend and divisor by the same number affects the quotient how? Multiplying both terms of a fraction by the same number affects the value of the fraction how?

1. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{2}{5}$ to fractions whose denominator is 120.

$\frac{1}{2} = \frac{60}{120}$. What must you do with the denominator 2 to make it 120? If you multiply 2 by 60, what must you do with the numerator so that the value of the fraction may not be changed?

$\frac{2}{3} = \frac{80}{120}$. By what must you multiply the terms of $\frac{2}{3}$ to reduce it to 120ths? How do you find that number to be 40?

$\frac{3}{4} = \frac{90}{120}$. By what must you multiply the terms of $\frac{3}{4}$ to make it $\frac{90}{120}$? How do you find that number to be 30?

$\frac{2}{5} = \frac{48}{120}$. By what must you multiply the terms of $\frac{2}{5}$ to make it $\frac{48}{120}$? How do you find that number to be 24?

Since the common denominator must be divided by the denominators of all the given fractions, it must be what of their denominators?

Then the least common denominator must be what of the given denominators?

These questions suggest the following

Rule for Reduction of Fractions to their Least Common Denominator

1. Find the least common multiple of the denominators of the given fractions. This is the least common denominator.

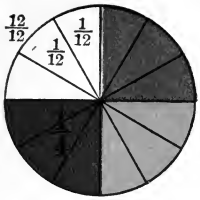
2. Divide the least common denominator by the denominator of the first of the given fractions. Multiply its numerator by the quotient obtained. The product is the required numerator.

3. Proceed in the same way with each of the given fractions.

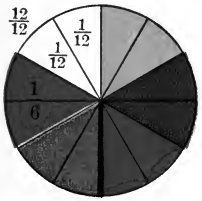
THE RULE ILLUSTRATED

2. Reduce $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{2}{3}$ to their least common denominators.

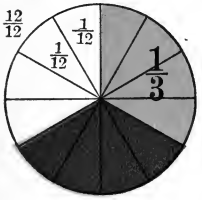
2 ×	4	6	3
3	2	3	3
	× 2	× 1	× 1 = 12, least common denominator.



How many 12ths in $\frac{1}{4}$? $(12 \div 4 = 3.)$
How many 12ths in $\frac{3}{4}$? $(\frac{3 \times 3}{4 \times 3} = \frac{9}{12}.)$



How many 12ths in $\frac{1}{6}$? $(12 \div 6 = 2.)$
How many 12ths in $\frac{5}{6}$? $(\frac{5 \times 2}{6 \times 2} = \frac{10}{12}.)$



How many 12ths in $\frac{1}{3}$? $(12 \div 3 = 4.)$
How many 12ths in $\frac{2}{3}$? $(\frac{2 \times 4}{3 \times 4} = \frac{8}{12}.)$

Change the following to fractions having a least common denominator:

146. Mental.

1. $\frac{1}{2}, \frac{3}{4}$
2. $\frac{2}{3}, \frac{5}{6}$
3. $\frac{1}{2}, \frac{7}{9}$
4. $\frac{5}{7}, \frac{1}{14}, \frac{1}{2}$
5. $\frac{1}{5}, \frac{1}{3}, \frac{7}{15}$
6. $\frac{1}{2}, \frac{2}{7}, \frac{1}{21}$
7. $\frac{2}{3}, \frac{5}{9}, \frac{7}{18}$
8. $\frac{5}{6}, \frac{3}{4}, \frac{2}{3}$
9. $\frac{6}{5}, \frac{5}{6}, \frac{1}{2}$

147. Written.

1. $\frac{1}{4}, \frac{3}{5}, \frac{5}{6}$
2. $\frac{2}{3}, \frac{4}{5}, \frac{4}{9}$
3. $\frac{5}{8}, \frac{9}{10}, \frac{1}{2}$
4. $\frac{7}{9}, \frac{2}{3}, \frac{3}{2}$
5. $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{5}{12}$
6. $\frac{1}{4}, \frac{3}{5}, \frac{5}{6}, \frac{7}{8}$
7. $9, \frac{5}{8}, \frac{9}{10}, \frac{4}{5}$
8. $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{12}$
9. $\frac{3}{4}, \frac{3}{5}, \frac{4}{9}, \frac{9}{10}$
10. $\frac{5}{8}, \frac{7}{10}, \frac{3}{4}, \frac{4}{5}$
11. $\frac{1}{2}, \frac{2}{5}, \frac{2}{3}, \frac{7}{15}$
12. $\frac{2}{5}, \frac{3}{8}, \frac{1}{4}, \frac{11}{12}$

ADDITION OF FRACTIONS

148. A proper fraction is in its **simplest form** when it is in its lowest terms.

An improper fraction is in its **simplest form** when it is reduced to an integer or a mixed number.

149. 1. Add $\frac{5}{12}$ and $\frac{3}{12}$. $\frac{5}{12}+\frac{3}{12}=\frac{5+3}{12}=\frac{8}{12}=\frac{2}{3}$. *Ans.*

2. Add $\frac{3}{4}$, $\frac{6}{4}$, $\frac{2}{4}$, and $\frac{1}{4}$.

$$\frac{3}{4}+\frac{6}{4}+\frac{2}{4}+\frac{1}{4}=\frac{3+6+2+1}{4}=\frac{12}{4}=3. \quad \textit{Ans.}$$

3. Add $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{16}$.

$$\frac{2}{3}+\frac{3}{4}+\frac{5}{16}=\frac{32}{48}+\frac{36}{48}+\frac{15}{48}=\frac{32+36+15}{48}=\frac{83}{48}=1\tfrac{35}{48}. \quad \textit{Ans.}$$

$$\begin{array}{r|l} 2 & \\ \times\,2 & 3\quad 2\quad 8 \end{array}$$

$$\times 3 \times 1 \times 4 = 48$$

Why do we reduce these fractions to their least common denominator?

150. The above examples illustrate the following

Rule for Addition of Fractions

1. Reduce the fractions to their least common denominator.
2. Add the new numerators, and write the sum over the least common denominator.
3. Reduce the result to its simplest form.

151. Addition of Mixed Numbers.

1. Add $3\frac{2}{5}$, $5\frac{1}{3}$ and $7\frac{3}{10}$.

$3\frac{2}{5}+5\frac{1}{3}+7\frac{3}{10}=3+5+7+\frac{12}{30}+\frac{10}{30}+\frac{9}{30}=15\frac{31}{30}=16\frac{1}{30}$. *Ans.*

152.

Rule for Addition of Mixed Numbers

1. Add the integers and fractions separately.
2. Unite the sum of the integers and the sum of the fractions.
3. Reduce the result to its simplest form.

153. Mental. Add the following:

1. $\frac{1}{2}$, $\frac{1}{3}$	**5.** $\frac{1}{4}$, $\frac{1}{5}$	**9.** $\frac{2}{7}$, $\frac{1}{3}$, 3	**13.** $1\frac{1}{4}$, $2\frac{1}{2}$, $\frac{1}{3}$
2. $\frac{2}{3}$, $\frac{1}{2}$	**6.** $\frac{3}{5}$, $\frac{1}{3}$	**10.** $\frac{1}{4}$, $\frac{1}{2}$, $1\frac{1}{2}$	**14.** $3\frac{1}{3}$, $1\frac{2}{5}$, 5
3. $\frac{2}{3}$, $\frac{1}{4}$	**7.** $\frac{1}{7}$, $\frac{1}{2}$	**11.** $2\frac{1}{3}$, $4\frac{1}{2}$, 1	**15.** $\frac{6}{7}$, $3\frac{1}{2}$, 4
4. $\frac{3}{4}$, $\frac{2}{3}$	**8.** $\frac{1}{3}$, $\frac{1}{6}$	**12.** $\frac{1}{7}$, $\frac{1}{2}$, $\frac{1}{14}$	**16.** $2\frac{1}{8}$, $3\frac{1}{4}$, $2\frac{1}{2}$

17. A man paid $\$\frac{3}{4}$ for a book, $\$\frac{7}{8}$ for an inkstand, and $\$\frac{1}{4}$ for writing paper. How much did he spend?

18. Mary has $\$\frac{2}{3}$; her mother gave her $\$3\frac{1}{2}$. How much has she?

154. Written. Addition:

1. $\frac{5}{7}$, $\frac{1}{14}$, $\frac{1}{2}$	**7.** $\frac{2}{5}$, $\frac{3}{10}$, $\frac{7}{20}$, $\frac{1}{2}$	**13.** $\frac{1}{2}$, $2\frac{2}{5}$, $\frac{2}{3}$, 6
2. $\frac{1}{5}$, $\frac{1}{3}$, $\frac{7}{15}$	**8.** $\frac{2}{3}$, $\frac{1}{8}$, $\frac{5}{12}$, $\frac{7}{24}$	**14.** $\frac{4}{5}$, $\frac{3}{8}$, $2\frac{1}{4}$, $\frac{7}{10}$
3. $\frac{2}{3}$, $\frac{5}{9}$, $\frac{7}{18}$	**9.** $\frac{1}{4}$, $\frac{3}{5}$, $\frac{5}{6}$, $\frac{7}{8}$	**15.** $4\frac{5}{9}$, $\frac{2}{7}$, $\frac{1}{3}$, $\frac{8}{21}$
4. $\frac{5}{6}$, $\frac{3}{4}$, $\frac{2}{3}$	**10.** $\frac{3}{4}$, $\frac{4}{5}$, 6, $\frac{4}{9}$	**16.** $\frac{3}{5}$, $\frac{1}{2}$, $\frac{9}{10}$, 4
5. $\frac{5}{12}$, $\frac{1}{2}$, $\frac{5}{4}$	**11.** $\frac{3}{4}$, $\frac{4}{9}$, $\frac{7}{11}$, $\frac{3}{5}$	**17.** $6\frac{3}{4}$, $8\frac{2}{3}$, $\frac{6}{7}$, $\frac{1}{8}$
6. $\frac{1}{8}$, $\frac{3}{4}$, $\frac{1}{2}$, $\frac{3}{16}$	**12.** $\frac{3}{8}$, $\frac{9}{10}$, $\frac{3}{4}$, $\frac{4}{5}$	**18.** 3, $\frac{2}{3}$, $\frac{1}{7}$, $\frac{3}{6}$

19. $\frac{5}{6}$, 4, $\frac{2}{3}$, $1\frac{4}{5}$ **21.** $6\frac{2}{3}$, $8\frac{1}{2}$, $5\frac{3}{4}$, $7\frac{5}{8}$ **23.** $\frac{5}{3}$, $\frac{5}{6}$, $\frac{6}{8}$, $\frac{2}{4}$,

20. $\frac{3}{5}$, $\frac{7}{6}$, $\frac{7}{10}$, $\frac{8}{15}$ **22.** $9\frac{1}{5}$, $5\frac{2}{7}$, 9, $\frac{1}{3}\frac{3}{5}$ **24.** $5\frac{1}{4}$, $7\frac{3}{8}$, $9\frac{1}{6}$, 45

25. $14\frac{3}{7}$, $9\frac{5}{8}$, $10\frac{1}{2}$, $12\frac{1}{1}\frac{3}{4}$

26. A man travels $25\frac{2}{3}$ miles on Monday, $37\frac{3}{5}$ miles on Tuesday, on Wednesday as many miles as on Monday and Tuesday. How many miles does he travel in three days?

27. A farmer has $27\frac{1}{2}$ bushels of potatoes in one bin, $133\frac{8}{9}$ bushels in another, $47\frac{5}{18}$ bushels in another. How many bushels has he?

28. Mr. Brown has $\$130\frac{5}{7}$; his wife $\$25\frac{5}{6}$ more than he has; his son $\$78\frac{9}{14}$, and his daughter $\$5\frac{2}{3}$ more than his son. How much have all?

29. How many yards of cloth will I have, if I buy $123\frac{7}{8}$ yards, $76\frac{3}{4}$ yards, and $58\frac{2}{3}$ yards?

30. $6\frac{1}{6}$ yards of cloth are required for a coat, $3\frac{5}{8}$ yards for trousers, and $\frac{7}{9}$ yards for a vest. How many yards are required?

31. Find what your mother spends if she pays $\$8\frac{7}{12}$ for your coat, $\$9\frac{1}{5}$ for your dress, $\$4\frac{2}{3}$ for your hat, $\$2\frac{1}{4}$ for your shoes, and $\$1\frac{1}{8}$ for your gloves.

32. How much land in a farm of five fields if the first contains $26\frac{1}{1}\frac{1}{2}$ acres, the second $50\frac{1}{2}\frac{6}{0}$ acres, the third $41\frac{6}{7}$ acres, the fourth $69\frac{3}{4}$ acres, and the fifth $52\frac{2}{3}$ acres?

SUBTRACTION OF FRACTIONS

155. 1. From $\frac{8}{9}$ take $\frac{7}{3}\frac{7}{0}$.

$$\begin{array}{l} 3\left|\dfrac{8}{9}-\dfrac{7}{30}=\dfrac{80}{90}-\dfrac{21}{90}=\dfrac{59}{90}.\right. \quad \textit{Ans.} \\ \times 3\times 10=90 \end{array}$$

Rule for Subtraction of Fractions

1. Reduce the fractions to their least common denominator.

2. Subtract the numerator of the subtrahend from that of the minuend and write the result over the least common denominator.

3. Reduce to simplest form.

SUBTRACTION OF MIXED NUMBERS

156. 1. From $10\frac{7}{18}$ take $3\frac{32}{45}$.

$10\frac{7}{18} = 10\frac{35}{90} = 9\frac{125}{90}$ $(1 = \frac{90}{90};\ \frac{35}{90} + \frac{90}{90} = \frac{125}{90};\ 10 - 1 = 9)$

$3\frac{32}{45} = 3\frac{64}{90} = 3\frac{64}{90}$

$6\frac{61}{90}$. *Ans.*

Rule for Subtraction of Mixed Numbers

1. Subtract the integers and fractions separately. If the fraction in the minuend is smaller than the fraction in the subtrahend, take 1 from the integer of the minuend and add its value to the fraction of the minuend, before subtracting.

157. Mental. Subtraction:

1. $\frac{3}{4} - \frac{1}{4}$	**5.** $\frac{9}{14} - \frac{2}{7}$	**9.** $7 - \frac{1}{2}$	**13.** $4 - 1\frac{1}{2}$
2. $\frac{3}{5} - \frac{1}{2}$	**6.** $\frac{5}{8} - \frac{3}{4}$	**10.** $8 - \frac{3}{5}$	**14.** $6\frac{1}{3} - 4\frac{2}{3}$
3. $\frac{5}{6} - \frac{1}{3}$	**7.** $\frac{9}{10} - \frac{3}{5}$	**11.** $12 - \frac{11}{12}$	**15.** $11 - 8\frac{2}{3}$
4. $\frac{1}{2} - \frac{1}{6}$	**8.** $\frac{19}{24} - \frac{3}{4}$	**12.** $22 - \frac{13}{15}$	**16.** $19\frac{1}{7} - 9\frac{5}{7}$

158. Written.

1. $\frac{2}{3} - \frac{1}{4}$	**6.** $15 - 2\frac{1}{2}$	**11.** $\frac{17}{18} - \frac{2}{9}$
2. $\frac{1}{2} - \frac{1}{3}$	**7.** $8\frac{1}{3} - 4\frac{1}{4}$	**12.** $\frac{3}{4} - \frac{1}{3}$
3. $\frac{7}{8} - \frac{1}{2}$	**8.** $10 - \frac{5}{7}$	**13.** $\frac{9}{10} - \frac{6}{7}$
4. $6 - \frac{1}{3}$	**9.** $12\frac{1}{4} - 5\frac{1}{2}$	**14.** $\frac{11}{15} - \frac{1}{7}$
5. $3\frac{2}{4} - \frac{3}{4}$	**10.** $8\frac{1}{2} - 4\frac{1}{4}$	**15.** $\frac{19}{29} - \frac{1}{8}$

16. Take $\frac{1}{5}$ from $\frac{5}{6}$. From $\frac{19}{20}$ take $\frac{9}{11}$.

17. From the sum of $\frac{5}{6}$ and $\frac{7}{8}$ take their difference.

18. What must I add to $9\frac{3}{4}$ to make $20\frac{8}{15}$?

19. The sum of two fractions is $\frac{17}{18}$; one of the fractions is $\frac{2}{5}$. What is the other?

MULTIPLICATION OF FRACTIONS

159. A fraction is an expression of what operation? The numerator of a fraction is which term in division? The denominator? The value of the fraction?

Principles

1. Multiplying the dividend multiplies the quotient. Therefore, multiplying the numerator multiplies the value of the fraction.

2. Multiplying the divisor divides the quotient. Therefore, multiplying the denominator divides the value of the fraction.

3. Dividing the dividend divides the quotient. Therefore, dividing the numerator divides the value of the fraction.

4. Dividing the divisor multiplies the quotient. Therefore, dividing the denominator multiplies the value of the fraction.

In what two ways may the value of a fraction be multiplied? Divided?

1. Multiply $\frac{5}{16}$ by 8.

$$\frac{5}{16} \times 8 = \frac{5 \times 8}{16} = \frac{40}{16} = 2\frac{8}{16} = 2\frac{1}{2}. \quad \textit{Ans.}$$

Or,
$$\frac{5}{16} \times 8 = \frac{5}{16 \div 8} = \frac{5}{2} = 2\frac{1}{2}. \quad \textit{Ans.}$$

State the principles applied in these operations.

160.

Rule for Multiplication of Fractions by Integers

1. Multiply the numerator of the fraction by the integer; or, divide the denominator of the fraction by the integer.
2. Reduce the result to its simplest form.

161. Mental. Multiply:

1. $\frac{3}{5}$ by 2	**5.** $\frac{4}{5}$ by 5	**9.** $\frac{3}{7}$ by 6	**13.** $\frac{4}{12}$ by 4
2. $\frac{5}{8}$ by 4	**6.** $\frac{6}{7}$ by 7	**10.** $\frac{3}{8}$ by 4	**14.** $\frac{7}{9}$ by 3
3. $\frac{2}{3}$ by 3	**7.** $\frac{8}{9}$ by 9	**11.** $\frac{7}{10}$ by 5	**15.** $\frac{3}{20}$ by 5
4. $\frac{5}{6}$ by 3	**8.** $\frac{3}{4}$ by 2	**12.** $\frac{2}{7}$ by 6	

16. At $\$\frac{3}{4}$ each, what will 8 books cost?

17. If a horse eat $\frac{3}{4}$ bushel of oats in a week, how much will he eat in 4 weeks?

18. If a pound of tea costs $\$\frac{2}{5}$, what will 6 pounds cost?

19. Multiply the following fractions by 3 and give the result in its simplest form: $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$, $\frac{8}{9}$.

Multiply the fractions in example 19 by 5; by 6; by 10.

162. Written. Multiply:

1. $13\frac{3}{14}$ by 7	**7.** $17\frac{7}{18}$ by 6	**13.** $14\frac{14}{50}$ by 10
2. $\frac{5}{9}$ by 6	**8.** $\frac{9}{20}$ by 10	**14.** $\frac{11}{12}$ by 12
3. $14\frac{4}{25}$ by 5	**9.** $18\frac{8}{35}$ by 7	**15.** $\frac{6}{15}$ by 11
4. $14\frac{4}{28}$ by 7	**10.** $17\frac{7}{42}$ by 6	**16.** $\frac{3}{4}$ by 20
5. $18\frac{8}{24}$ by 9	**11.** $\frac{3}{45}$ by 15	**17.** $21\frac{21}{33}$ by 18
6. $13\frac{3}{15}$ by 8	**12.** $\frac{3}{16}$ by 25	**18.** $17\frac{17}{19}$ by 26

19. $\frac{3}{4}$ of $12 = ?$

SOLUTION. — $\frac{1}{4}$ of $12 = \frac{12}{4} = 3$.

$\frac{3}{4}$ of $12 = 3 \times 3 = 9$. *Ans.*

20. $\frac{5}{8}$ of $7 = ?$

SOLUTION. — $\frac{1}{8}$ of $7 = \frac{7}{8}$.

$\frac{5}{8}$ of 7 = 5 times $\frac{7}{8} = \frac{35}{8} = 4\frac{3}{8}$. *Ans.*

NOTE. — $\frac{5}{8}$ *of* 7 is the same as $\frac{5}{8}$ *times* 7, and the product is the same as 7 times $\frac{5}{8}$.

21. $\frac{3}{7}$ of $5 = ?$ $\frac{6}{11}$ of $4 = ?$ $\frac{5}{6}$ of $3 = ?$ $\frac{2}{3}$ of $6 = ?$

22. \$10 multiplied by $\frac{7}{8} = ?$ $\frac{3}{40} \times 10 = ?$

23. $\frac{3}{5} \times 8 = ?$ $4 \times \frac{6}{7} = ?$ $\frac{6}{7} \times 4 = ?$

24. Multiply $18\frac{3}{4}$ by 8, first reducing the mixed number to an improper fraction.

25. Multiply $18\frac{3}{4}$ by 8 without reducing the mixed number to an improper fraction. First multiply the fraction, then the integer, by 8, and add the products. Thus:

$$\begin{array}{r} 18\frac{3}{4} \\ \hline 8 \\ \hline 6 \\ 144 \\ \hline 150 \end{array}$$

SOLUTION. —

8 times $\frac{3}{4} = \frac{24}{4} = 6$

8 times 18 = 144

150

26. $16\frac{4}{5}$ multiplied by 10.	**29.** Multiply $27\frac{14}{17}$ by 38.
27. $126\frac{5}{9}$ multiplied by 9.	**30.** Multiply $35\frac{12}{92}$ by 46.
28. $326\frac{7}{8}$ multiplied by 5.	**31.** 9 times $21\frac{24}{135} = ?$

32. Multiply 47 by $6\frac{3}{4}$, first reducing the mixed number to an improper fraction.

33. Multiply 65 by $7\frac{2}{3}$.

34. Multiply 125 by $14\frac{5}{8}$.

35. $4\frac{7}{8}$ times 48 = ?

36. $19\frac{3}{5}$ times 385 = ?

163. **1.** Multiply $\frac{2}{3}$ by $\frac{5}{7}$.

Since $\frac{5}{7}$ means 5 divided by 7, then to multiply $\frac{2}{3}$ by $\frac{5}{7}$ is to multiply it by 5 and divide it by 7. What is the easiest way to multiply $\frac{2}{3}$ by 5? $\left(\frac{2 \times 5}{3}\right)$. To divide it by 7? $\left(\frac{2 \times 5}{3 \times 7}\right)$. See principles, Article 164. Therefore

$$\frac{2}{3} \times \frac{5}{7} = \frac{2 \times 5}{3 \times 7} = \frac{10}{21}. \quad \textit{Ans.}$$

2. $\frac{24}{35} \times \frac{55}{6} = ?$ $\quad \frac{\overset{2}{\cancel{24}}}{\underset{7}{\cancel{35}}} \times \frac{\overset{11}{\cancel{55}}}{\underset{3}{\cancel{6}}} = \frac{22}{21} = 1\frac{1}{21}$. *Ans.*

3. $\frac{3}{5} \times \frac{10}{27} \times \frac{9}{17} \times \frac{15}{24} = ?$ $\quad \frac{\overset{}{\cancel{3}}}{\underset{}{\cancel{5}}} \times \frac{\overset{2}{\cancel{10}}}{\underset{9}{\cancel{27}}} \times \frac{\overset{}{\cancel{9}}}{\underset{}{\cancel{17}}} \times \frac{\overset{5}{\cancel{15}}}{\underset{4}{\cancel{24}}} = \frac{5}{68}$. *Ans.*

How was the work shortened in examples 2 and 3?

164. These examples illustrate the following

Rule for Multiplication of a Fraction by a Fraction

1. Multiply all the numerators together.
2. Multiply all the denominators together.
3. Cancel when possible.
4. Write the first product over the second and reduce to simplest form.

NOTE 1. — Any integer may be expressed as a fraction by writing it over the denominator 1.

NOTE 2. — The word *of* between fractions means the same as the sign of multiplication.

Fractions joined by *of* form a **Compound Fraction.**

NOTE 3. — Mixed numbers must be changed to improper fractions before multiplying them together.

Oral.

1. How much is $\frac{1}{2}$ of $\frac{1}{4}$ of an inch?
2. Illustrate that $\frac{1}{4}$ of $\frac{1}{2}$ of an apple is $\frac{1}{8}$ of an apple.
3. Multiply $\frac{3}{5}$ by $\frac{1}{4}$; $\frac{4}{5}$ by $\frac{1}{2}$; $\frac{1}{2}$ by $\frac{1}{3}$; $\frac{1}{6}$ by $\frac{1}{2}$.
4. How much is $\frac{1}{4}$ of $\frac{4}{7}$? $\frac{3}{5}$ of $\frac{5}{7}$? $\frac{1}{4}$ of $\frac{4}{5}$? $\frac{1}{6}$ of $\frac{6}{11} = ?$
5. A man owned $\frac{3}{4}$ of a farm and sold $\frac{1}{2}$ of his share. What part of the farm did he sell?
6. James had \$ $\frac{3}{4}$, and John $\frac{1}{4}$ as much. How much had both?
7. If a pound of tea costs \$ $\frac{3}{8}$, what will $\frac{1}{4}$ pound cost?

165. Written. Find the products:

1. $\frac{5}{6} \times \frac{7}{8}$
2. $\frac{3}{4} \times \frac{4}{5}$
3. $\frac{9}{10} \times \frac{5}{8}$
4. $\frac{7}{9}$ of $\frac{9}{14}$
5. $\frac{5}{6}$ of $\frac{12}{20}$
6. $\frac{7}{19} \times \frac{9}{14}$
7. $\frac{3}{4}$ of $\frac{1}{2}$ of $\frac{2}{3}$
8. $\frac{5}{6}$ of $\frac{9}{10}$ of $\frac{4}{5}$
9. $\frac{4}{9} \times \frac{3}{12} \times \frac{6}{7}$
10. $\frac{5}{16} \times \frac{8}{9} \times \frac{3}{4}$
11. $\frac{9}{25} \times \frac{10}{7} \times \frac{8}{9}$
12. $\frac{12}{17} \times 34 \times \frac{5}{8}$
13. $5\frac{3}{4} \times 2\frac{4}{5} \times 20$
14. $7\frac{1}{2} \times 5\frac{4}{9} \times \frac{3}{7}$
15. $9\frac{1}{4} \times \frac{4}{37} \times 2\frac{1}{5}$
16. $6\frac{1}{4} \times \frac{11}{25} \times \frac{5}{11}$
17. $\frac{7}{16} \times 4 \times 5\frac{1}{2}$
18. $\frac{9}{10} \times 80 \times 5\frac{1}{2}$

Find the value:

19. $\frac{4}{5}$ of 40
20. $\frac{6}{7}$ of 42
21. $\frac{3}{5}$ of 16
22. $\frac{5}{8}$ of 328
23. $\frac{3}{7}$ of 721
24. $\frac{4}{5}$ of 90
25. $\frac{17}{18}$ of 342
26. $\frac{15}{16}$ of 800
27. $\frac{9}{11}$ of 2222

28. $\frac{7}{8}$ of $\frac{5}{9}$ of $\frac{6}{7}$ of $\frac{2}{3}$ of $\frac{4}{5} = ?$ $\frac{9}{10}$ of $\frac{6}{7}$ of $\frac{11}{2}$ of $\frac{6}{11} = ?$

29. $\frac{6}{11}$ of $\frac{33}{12}$ of $\frac{9}{10}$ of $\frac{15}{20}$ of $\frac{2}{3} = ?$ $\frac{7}{8}$ of $\frac{16}{21}$ of $\frac{4}{5}$ of $\frac{25}{28} = ?$

30. Mr. Brown earns $\$40\frac{3}{4}$ a month, and his son $\frac{7}{8}$ as much. How much does the son earn?

31. At $\$12\frac{5}{8}$ a ton, how much will $9\frac{3}{20}$ tons of hay cost?

32. What will be the cost of $48\frac{3}{4}$ yards of cloth at $\$\frac{5}{8}$ a yard?

33. A man gave $124\frac{5}{16}$ acres of land to his two sons, giving $\frac{3}{5}$ of it to the elder and $\frac{2}{5}$ to the younger. How many acres did each receive?

34. If it requires $21\frac{3}{4}$ days for a man to dig a ditch, in what time can he dig $\frac{5}{7}$ of it?

DIVISION OF FRACTIONS

166. What operations upon the dividend and divisor divide the quotient?

What operations on the numerator and denominator divide the value of the fraction?

1. $\frac{9}{17} \div 3 = ?$ What is the easiest way to divide $\frac{9}{17}$ by 3?
$\frac{9}{17} \div 3 = \frac{9 \div 3}{17} = \frac{3}{17}$. *Ans.*

2. $\frac{7}{19} \div 3 = ?$ What is the easiest way to divide $\frac{7}{19}$ by 3?
$\frac{7}{19} \div 3 = \frac{7}{19 \times 3} = \frac{7}{57}$. *Ans.*

167.

Rule for Dividing a Fraction by an Integer

1. Divide the numerator or multiply the denominator of the fraction by the given integer.
2. Reduce the result to its simplest form.

168. Mental. Divide:

3. $\frac{3}{4}$ by 3	**7.** $\frac{15}{16}$ by 5	**11.** $\frac{15}{24}$ by 8	**15.** $\frac{15}{25}$ by 5
4. $\frac{2}{3}$ by 3	**8.** $\frac{3}{4}$ by 6	**12.** $\frac{7}{50}$ by 10	**16.** $\frac{3}{5}$ by 3
5. $\frac{8}{9}$ by 4	**9.** $\frac{6}{7}$ by 3	**13.** $\frac{16}{32}$ by 8	**17.** $\frac{5}{10}$ by 4
6. $\frac{7}{9}$ by 4	**10.** $\frac{7}{9}$ by 3	**14.** $\frac{5}{14}$ by 7	**18.** $\frac{16}{5}$ by 4

19. A man divides $\$\frac{5}{8}$ equally among 4 boys. What part of a dollar does each receive?

20. A boy wishes to put $\frac{5}{8}$ of a bushel of chestnuts into 5 bags. How much will each bag contain?

21. If 4 pounds of coffee cost $\$\frac{2}{3}$, what will one pound cost?

22. Divide $\$\frac{1}{2}$ equally among 5 boys. What is the share of each?

23. How much is $\frac{1}{3} \div 4$? $\frac{1}{5}$ divided by 5? $\frac{1}{6}$ divided by 6?

169. Written. Divide:

1. $\frac{15}{16}$ by 5	**6.** $\frac{24}{16}$ by 7	**11.** $\frac{15}{6}$ by 3
2. $\frac{24}{32}$ by 6	**7.** $\frac{13}{18}$ by 2	**12.** $\frac{18}{24}$ by 6
3. $\frac{20}{25}$ by 5	**8.** $\frac{11}{25}$ by 3	**13.** $\frac{46}{52}$ by 2
4. $\frac{3}{17}$ by 7	**9.** $\frac{40}{52}$ by 10	**14.** $\frac{35}{42}$ by 7
5. $\frac{15}{8}$ by 4	**10.** $\frac{25}{30}$ by 5	**15.** $\frac{84}{35}$ by 12

16. Divide $14\frac{2}{3}$ by 3.

$14\frac{2}{3} = \frac{44}{3}$

$\frac{44}{3} \div 3 = \frac{44}{9} = 4\frac{8}{9}$

Or,

$$3\overline{)14\frac{2}{3}}$$

$$4\frac{8}{9}$$

$2\frac{2}{3} \div 3 = \frac{8}{3} \div 3 = \frac{8}{9}$

SOLUTION. — We may change the mixed number to an improper fraction and divide according to the rule, or we may divide without changing to an improper fraction. $14\frac{2}{3} \div 3 = 4$ with a remainder of $2\frac{2}{3}$. $2\frac{2}{3} = \frac{8}{3}$. $\frac{8}{3} \div 3 = \frac{8}{9}$.

Find the quotients:

17. $47\frac{6}{7} \div 7$ **20.** $385\frac{4}{5} \div 5 = ?$ **23.** $264\frac{3}{7} \div 4 = ?$

18. $384\frac{3}{5} \div 5$ **21.** $16\frac{4}{5} \div 5 = ?$ **24.** $9826\frac{2}{13} \div 6 = ?$

19. $.287\frac{9}{10} \div 8$ **22.** $30\frac{5}{7} \div 7 = ?$

25. If 16 bushels of apples cost $\$8\frac{7}{8}$, what will 1 bushel cost?

26. Five heirs shared equally in the division of a legacy of $\$35{,}862\frac{5}{8}$. What was the share of each?

170. **1.** $\frac{3}{5} \div \frac{7}{11} = ?$ $\frac{7}{11} = (7 \div 11)$.

$\frac{3}{5} \div \frac{7}{11} = \frac{3}{5} \div (7 \div 11)$.

$\frac{3}{5} \div 7 = \frac{3}{5 \times 7}$. What principle is applied here? We have divided $\frac{3}{5}$ by 7. The divisor given was $\frac{7}{11}$; 7 is how many times as large as $\frac{7}{11}$? Since we have used a divisor 11 times as large as the given divisor, how does the quotient compare with the correct quotient? State the principle. How shall we correct the quotient? What is the easiest way to multiply $\frac{3}{5 \times 7}$ by 11? The quotient then becomes $\frac{3 \times 11}{5 \times 7}$. This is the same as multiplying $\frac{3}{5}$ by what fraction? $\frac{3}{5} \times \frac{11}{7} = \frac{33}{35}$. *Ans.*

What do you do with $\frac{7}{11}$ to get $\frac{11}{7}$?

2. $19 \div \frac{8}{13} = ?$ $19 = \frac{19}{1}$. $\frac{19}{1} \div \frac{8}{13} = \frac{19}{1} \times \frac{13}{8} = \frac{247}{8} = 30\frac{7}{8}$. *Ans.*

What is done with the integer before dividing?

3. $6\frac{11}{18} \div \frac{17}{64} = ?$ $6\frac{11}{18} \div \frac{17}{64} = \frac{\overset{7}{\cancel{119}}}{\underset{9}{\cancel{18}}} \times \frac{32}{\cancel{64}\,\cancel{17}} = \frac{224}{9} = 24\frac{8}{9}$. *Ans.*

What is done with the mixed number before dividing?

171. The above examples illustrate the following

Rule for the Division of a Fraction by a Fraction

1. Interchange the terms of the divisor.
2. Multiply the dividend by the divisor with terms inverted.
3. Express integers and mixed numbers as fractions before dividing.

172. Example 1 may be solved also by reducing both dividend and divisor to a common denominator, thus:

$$\frac{3}{5} \div \frac{7}{11} = \frac{3 \times 11}{5 \times 11} \div \frac{7 \times 5}{11 \times 5}.$$

The denominator 5×11 being common to both fractions, we divide the numerator of the dividend by the numerator of the divisor, thus,

$$\frac{3 \times 11}{7 \times 5} = \frac{33}{35}.$$

This gives the same result as multiplying the dividend by the divisor with terms interchanged, thus, $\frac{3}{5} \times \frac{11}{7} = \frac{33}{35}$.

173. Find the quotients:

4. $\frac{7}{8} \div \frac{3}{4}$

5. $\frac{11}{12} \div \frac{5}{6}$

6. $\frac{5}{18} \div \frac{3}{4}$

7. $\frac{16}{27} \div \frac{2}{3}$

8. $\frac{14}{15} \div \frac{7}{10}$

9. $3\frac{1}{2} \div \frac{14}{15}$

10. $5\frac{1}{4} \div \frac{9}{20}$

11. $\frac{3}{11} \div 5\frac{3}{4}$

12. $\frac{7}{10} \div 4\frac{1}{5}$

13. $\frac{13}{15} \div 5\frac{1}{2}$

14. $2 \div \frac{4}{5}$

15. $8 \div \frac{7}{10}$

16. $10 \div \frac{5}{6}$

17. $\frac{7}{8} \div 14$

18. $\frac{16}{21} \div 8$

19. $2\frac{3}{4} \div 5\frac{1}{2}$

20. $7\frac{1}{8} \div 1\frac{1}{2}$

21. $2\frac{3}{4} \div \frac{11}{12}$

22. $2\frac{1}{3} \div 3\frac{1}{2}$

23. $8\frac{1}{9} \div 9\frac{1}{7}$

24. $\frac{4}{5} \times \frac{3}{8} \div \frac{5}{6}$ of $\frac{3}{5} = ?$

SOLUTION. — Inverting the divisor, indicating the operations, and cancelling, we have

$$\frac{\overset{1}{\cancel{4}}}{5} \times \frac{3}{\underset{2}{\cancel{8}}} \times \frac{\overset{2}{\cancel{6}}}{\underset{1}{\cancel{5}}} \times \frac{\underset{1}{\cancel{5}}}{\underset{1}{\cancel{3}}} = \frac{3}{5}. \quad Ans.$$

25. $\frac{3}{4}$ of $9 \div \frac{5}{6}$ of $\frac{6}{7}$

26. $\frac{4}{5}$ of $1\frac{0}{12} \div \frac{5}{8}$ of 4

27. $3\frac{1}{2} \div \frac{4}{5} \times \frac{7}{8}$ of 2

28. $\frac{4}{5} \times \frac{5}{6} \times \frac{3}{4} \div 7\frac{1}{2}$

29. Divide 3682 by $5\frac{1}{2}$.

SOLUTION. — When the dividend contains several figures and the divisor is a mixed number, it is often more convenient to divide as above.

$$\begin{array}{r} 5\frac{1}{2})3682 \\ 2 \quad\quad 2 \\ \hline 11\ )\overline{7364} \\ \quad 669\frac{5}{11} \end{array}$$

We multiply both dividend and divisor by 2, when the divisor becomes 11 (halves), and the dividend 7364 (halves). Dividing, the quotient is $669\frac{5}{11}$.

Find the quotients:

30. $356 \div 4\frac{1}{2}$

31. $728 \div 8\frac{1}{5}$

32. $397 \div 5\frac{1}{4}$

33. $296 \div 10\frac{1}{2}$

34. $39{,}846 \div 3\frac{1}{5}$

35. $44{,}077 \div 5\frac{1}{2}$

36. $76{,}582 \div 9\frac{1}{4}$

37. $28{,}769 \div 7\frac{5}{8}$

38. There are $5\frac{1}{2}$ yards in a rod. How many rods in 3158 yards?

39. If a man walks $15\frac{1}{2}$ miles a day, in how many days can he walk 155 miles?

40. What is the price of coal per ton when 16 tons cost $\$73\frac{3}{5}$?

41. How much does a man earn in a day if he earns $\$84\frac{1}{2}$ in a month of 26 days?

42. When flour is $\$6\frac{3}{4}$ per barrel, how many barrels can be bought for $\$297$?

174. A fraction that has a fraction in either or both of its terms is a **Complex Fraction.** Thus,

$$\frac{3}{8\frac{2}{5}},\ \frac{\frac{2}{3}}{16},\ \frac{5\frac{1}{2}}{25},\ \frac{3\frac{2}{3}}{7\frac{4}{9}},\ \text{and}\ \frac{\frac{2}{3}\div 9}{1\frac{3}{8}-\frac{2}{5}}$$

are complex fractions.

A fraction whose terms are integers is a **Simple Fraction.** Thus, $\frac{11}{12}$ is a simple fraction.

1. Reduce $\dfrac{7}{8\frac{2}{3}}$ to a simple fraction.

$$\frac{7}{8\frac{2}{3}} = 7 \div 8\tfrac{2}{3} = \frac{7}{1} \div \frac{26}{3} = \frac{7}{1} \times \frac{3}{26} = \frac{21}{26}. \quad \textit{Ans.}$$

2. Reduce $\dfrac{\frac{5}{17}}{40}$ to a simple fraction.

$$\frac{\frac{5}{17}}{40} = \frac{5}{17} \div 40 = \frac{\cancel{5}}{17 \times \cancel{40}_{8}} = \frac{1}{136}. \quad \textit{Ans.}$$

3. Reduce $\dfrac{7\frac{5}{8}}{2\frac{13}{20}}$ to its simplest form.

$$\frac{7\frac{5}{8}}{2\frac{13}{20}} = 7\tfrac{5}{8} \div 2\tfrac{13}{20} = \frac{61}{8} \div \frac{53}{20} = \frac{61}{\cancel{8}_{2}} \times \frac{\cancel{20}^{5}}{53} = \frac{305}{106} = 2\tfrac{93}{106}. \quad \textit{Ans.}$$

175. From the examples we may obtain the following

Rule for the Reduction of a Complex Fraction to its Simplest Form

1. Perform the operations indicated in the numerator, if there are any.
2. Perform the operations indicated in the denominator, if there are any.
3. Divide the numerator by the denominator, and reduce the result to its simplest form.

176. Change to simple fractions:

4. $\dfrac{7\frac{1}{9}}{\frac{16}{27}}$ **6.** $\dfrac{15\frac{3}{4}}{4}$ **8.** $\dfrac{19}{16\frac{4}{5}}$ **10.** $\dfrac{3\frac{1}{2}}{\frac{7}{10}}$ **12.** $\dfrac{\frac{1}{4}\text{ of }\frac{7}{8}}{\frac{2}{3}\text{ of }\frac{5}{6}}$

5. $\dfrac{18\frac{1}{2}}{\frac{4}{5}}$ **7.** $\dfrac{\frac{17}{18}}{16}$ **9.** $\dfrac{\frac{7}{8}}{\frac{9}{16}}$ **11.** $\dfrac{18}{\frac{9}{16}}$ **13.** $\dfrac{\frac{2}{3}\text{ of }5\frac{1}{2}}{\frac{7}{8}}$

14. If $\frac{3}{5}$ of an acre of land is worth $\$72\frac{3}{10}$, what is the value of an acre at the same rate?

15. There are $5\frac{1}{2}$ yards in a rod. How many rods in $70\frac{1}{8}$ yards?

16. At $\$5\frac{1}{4}$ a ton, how many tons of coal can be bought for $\$73\frac{1}{2}$?

THE THREE QUESTIONS OF RELATION

177. 1. 3 times 4 equals what? *Ans.* 12.

2. 12 is how many times 4? *Ans.* 3.

3. 12 is 3 times what? *Ans.* 4.

In question 1, we have two factors, to find their product. In questions 2 and 3, we have the product and one factor, to find the other factor.

1. Form questions like 2 and 3 from the following statement: $5 \times 6 = 30$.

a. $\frac{1}{2}$ of 8 equals what?
Multiplying 8 by $\frac{1}{2}$, we have 4. *Ans.*

b. 4 is $\frac{1}{2}$ of what?
Since $\frac{1}{2} \times 8 = 4$, $4 \div \frac{1}{2} = 8$. *Ans.*

c. 4 is what part of 8?
Since $\frac{1}{2} \times 8 = 4$, $4 \div 8 = \frac{1}{2}$. *Ans.*

Principle

The product of two numbers divided by one of them gives the other.

To the Teacher. — In such examples as question *a*, after the product is found, it may be used with each of the two numbers to form successively, question *b* and question *c*. Drill upon these three questions of relation should be so thorough that each question will suggest its own solution instantly.

2. $\frac{1}{3}$ of 24 = what? (Question *a*.)

3. After finding the product in example 2, form question *b*. Question *c*.

4. 8 is $\frac{1}{3}$ of what? (Question *b*.)

Solution. — From the question it is evident that 8 is the product of two numbers, and that $\frac{1}{3}$ is one of them. Therefore, $8 \div \frac{1}{3} = 24$. 8 is $\frac{1}{3}$ of 24.

5. What part of 24 is 8? (Question *c*.)

Solution. — It is evident that 8 is the product of two numbers, and 24 is one of them. Therefore, $8 \div 24 = \frac{8}{24}$ or $\frac{1}{3}$. 8 is $\frac{1}{3}$ of 24.

Question *a*

Find the result, and form questions *b* and *c*:

6. How much is $\frac{3}{4}$ of 12?

7. How much is $\frac{5}{8}$ of 16?

8. How much is $\frac{4}{5}$ of 20?

9. $\frac{3}{5}$ of 15 = ?

10. $\frac{4}{7}$ of 21 = ?

11. $\frac{5}{8}$ of 40 = ?

Question *b*

Find the result, and form questions *a* and *c*:

12. 15 is $\frac{3}{4}$ of what?

13. 4 is $\frac{2}{3}$ of what?

14. 9 is $\frac{3}{5}$ of what?

15. 18 is $\frac{6}{7}$ of what?

16. 24 is $\frac{4}{5}$ of what?

17. 25 is $\frac{5}{8}$ of what?

Question *c*

Find the result, and form questions *a* and *b*:

18. What part of 24 is 8?
19. What part of 18 is 12?
20. What part of 9 is 2?
21. 21 is what part of 35?
22. 28 is what part of 63?
23. 15 is what part of 25?

Find the result, form the other two questions, and solve each:

24. $\frac{7}{8}$ of 56 equals what?
25. What part of 49 is 14?
26. 26 is $\frac{2}{3}$ of what?
27. 65 is what part of 120?
28. How much is $\frac{7}{24}$ of 96?
29. 38 is $\frac{2}{11}$ of what number?
30. 16 is what part of 80?
31. 18 is $\frac{9}{10}$ of what number?

REMARK. — Each of the following problems contains one or more of the three questions of relation. Before attempting to solve any of them, the pupil should state the question in each of them.

32. James had 56 marbles, and John $\frac{3}{4}$ as many. How many had John?

The question is, How much is $\frac{3}{4}$ of 56? — *a*.

33. John had 42 marbles, which was $\frac{3}{4}$ as many as James had. How many had James?

The question is, 42 is $\frac{3}{4}$ of what? — *b*.

34. James had 56 marbles, and John 42. John's marbles are what part of James's?

The question is, What part of 56 is 42? — *c*.

35. A man sold 50 acres of land, which was $\frac{5}{7}$ of all he had. How many acres had he at first?

36. A boy had 20 cents and spent 15 cents. What part of his money did he spend? What part was left?

37. Mr. A has 640 sheep, and Mr. B $\frac{7}{16}$ as many. How many has Mr. B?

ALIQUOT PARTS

178. An aliquot part of a number is any integer or mixed number that is exactly contained in it.

PARTS OF A DOLLAR

$6\frac{1}{4}$ cents $= \$\frac{1}{16}$	$37\frac{1}{2}$ cents $= \$\frac{3}{8}$
$8\frac{1}{3}$ cents $= \$\frac{1}{12}$	50 cents $= \$\frac{1}{2}$
$12\frac{1}{2}$ cents $= \$\frac{1}{8}$	$62\frac{1}{2}$ cents $= \$\frac{5}{8}$
$16\frac{2}{3}$ cents $= \$\frac{1}{6}$	$66\frac{2}{3}$ cents $= \$\frac{2}{3}$
25 cents $= \$\frac{1}{4}$	75 cents $= \$\frac{3}{4}$
$33\frac{1}{3}$ cents $= \$\frac{1}{3}$	$87\frac{1}{2}$ cents $= \$\frac{7}{8}$

What is the cost of 33 books at $16\frac{2}{3}¢$ each?

33 books cost 33 times $16\frac{2}{3}¢$, or 33 times $\$\frac{1}{6} = \$\frac{33}{6} =$ $5.50. *Ans.*

Oral. Multiply:

1. $12\frac{1}{2}$ cents by 16
2. $16\frac{2}{3}$ cents by 12
3. 25 cents by 20
4. $33\frac{1}{3}$ cents by 27
5. $6\frac{1}{4}$ cents by 16
6. $8\frac{1}{3}$ cents by 24
7. $37\frac{1}{2}$ cents by 8
8. 50 cents by 15
9. $62\frac{1}{2}$ cents by 8
10. $66\frac{2}{3}$ cents by 9
11. 75 cents by 4
12. $87\frac{1}{2}$ cents by 8

13. What is the cost of:

16 pounds of bacon @ $12\frac{1}{2}¢$ a pound?
16 balls @ $50¢$ each?
36 yards of ribbon @ $33\frac{1}{3}¢$ a yard?
36 pounds of candy @ $25¢$ a pound?
8 pounds of tea @ $62\frac{1}{2}¢$ a pound?

The sign @ means *at*. The sign ¢ means *cents*.

Written.

What is the cost of :

14. 66 pounds of pork at $12\frac{1}{2}$ cents a pound?
48 pounds of veal at $16\frac{2}{3}$ cents a pound?
65 boxes of strawberries at 25 cents a box?
15 yards of flannel at $33\frac{1}{3}$ cents a yard?
80 pounds of honey at 25 cents a pound?
48 pounds of tea at 50 cents a pound?

179. **1.** At 25¢ a pound, how many pounds of butter can be bought for $8?

As many pounds as 25¢ or $\$\frac{1}{4}$ is contained times in $8. $\$8 \div \$\frac{1}{4} = 32$ pounds. *Ans.*

Oral. Divide:

2. $3 by $33\frac{1}{3}$¢
3. $5 by 25¢
4. $2 by $8\frac{1}{3}$¢
5. $9 by $12\frac{1}{2}$¢
6. $1 by $6\frac{1}{4}$¢
7. $10 by 50¢
8. $6 by $33\frac{1}{3}$¢
9. $4 by $12\frac{1}{2}$¢
10. $2 by 25¢
11. $3 by $8\frac{1}{3}$¢

180. Written.

12. At 25 cents apiece, how many hats can be bought for $6?

13. At 25 cents a pound, how many pounds of cheese can be bought for $5?

14. At $16\frac{2}{3}$ cents a dozen, how many dozen eggs can be bought for $4?

15. How many pounds of beef can be bought for $4 at $.16\frac{2}{3}$ a pound?

16. At $33\frac{1}{3}$¢ a yard, how many yards of linen can be bought for $10?

17. How many penknives can be bought for $6 at $33\frac{1}{3}$ cents apiece?

18. $24 \times \$.12\frac{1}{2} = ?$

19. $\$24 \div \$.12\frac{1}{2} = ?$

REVIEW OF FRACTIONS

181. Oral.

1. Change $\frac{1}{3}$ to *sixths*. To *ninths*.

2. Change $\frac{2}{3}$ to *sixths*. To *ninths*.

3. Express $\frac{6}{8}$ in larger terms. What operations did you perform?

Express $\frac{6}{8}$ in smaller terms. What did you do?

Change the following:

4. $\frac{3}{5}$ to 10ths **6.** $\frac{3}{9}$ to 27ths **8.** $\frac{4}{5}$ to 25ths

5. $\frac{2}{3}$ to 9ths **7.** $\frac{7}{8}$ to 56ths **9.** $\frac{3}{7}$ to 84ths

Change to lowest terms:

10. $\frac{3}{6}$ **12.** $\frac{15}{18}$ **14.** $\frac{36}{72}$ **16.** $\frac{15}{21}$

11. $\frac{10}{12}$ **13.** $\frac{20}{30}$ **15.** $\frac{24}{40}$ **17.** $\frac{18}{24}$

18. Reduce $5\frac{1}{2}$ to halves. $7\frac{1}{8}$ to eighths. $4\frac{5}{6}$ to sixths. $4\frac{5}{7}$ to sevenths.

19. Change $4\frac{1}{9}$ to 9ths. $3\frac{2}{3}$ to 3ds. $5\frac{6}{10}$ to 10ths. $8\frac{4}{5}$ to 5ths. $7\frac{3}{11}$ to 11ths.

Reduce to improper fractions:

20. $3\frac{2}{3}$ **22.** $2\frac{5}{6}$ **24.** $8\frac{9}{10}$ **26.** $12\frac{7}{8}$

21. $4\frac{1}{2}$ **23.** $3\frac{5}{7}$ **25.** $9\frac{5}{6}$ **27.** $5\frac{11}{12}$

28. How many dollars in $\$\frac{25}{5}$? in $\$\frac{14}{7}$? in $\$\frac{28}{4}$?

29. 12 fourths of a bushel are equal to how many bushels? 36 fourths? 40 fourths?

30. To how many dollars are 8 fourths of a dollar equal? 9 fourths? 11 fourths?

Reduce to integers or mixed numbers:

31. $\frac{6}{2}$ **33.** $\frac{7}{3}$ **35.** $\frac{40}{8}$ **37.** $\frac{91}{9}$

32. $\frac{5}{4}$ **34.** $\frac{24}{3}$ **36.** $\frac{51}{3}$ **38.** $\frac{124}{2}$

39. Change $\frac{1}{4}$ and $\frac{1}{2}$ so both may have 20 for a denominator.

40. Change $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ each to 12ths.

41. Change $\frac{2}{3}$ and $\frac{3}{4}$ each to 24ths.

42. A boy paid $\frac{3}{8}$ of a dollar for a book and $\frac{1}{4}$ of a dollar for a paper. How much did he pay?

43. John saves $\frac{1}{2}$ a dollar a week, and Charles $\frac{3}{4}$ of a dollar. How many fourths do both save?

44. Henry gave $\frac{1}{4}$ of his marbles to one boy and $\frac{2}{3}$ of them to another. How many twelfths do both receive?

45. A clerk sold $\frac{1}{2}$ a pound of tea to one customer, $\frac{1}{4}$ to another, and $\frac{5}{8}$ to another. How many eighths did he sell?

46. A man pays $\frac{1}{5}$ of his salary for rent, $\frac{1}{4}$ for table expenses, and $\frac{2}{10}$ for clothing. What part of his money was expended for rent, table, and clothing?

47. A boy had $\$\frac{9}{10}$ and spent $\$\frac{1}{2}$. What part of a dollar had he left?

48. A owns $\frac{5}{8}$ of a store, and B $\frac{1}{4}$. How much of the store does A own more than B?

49. John runs $\frac{1}{2}$ of a mile, and Jerry $\frac{3}{8}$ of a mile. Which runs farther, and what part of a mile?

50. Mr. Ames owned $\frac{3}{4}$ of a farm and sold $\frac{1}{6}$ of it. What part remained?

51. What is the difference between $\frac{5}{6}$ of anything and $\frac{4}{5}$ of it? Which is greater?

52. Lucy has $\$\frac{1}{5}$, and Alice $\$\frac{3}{4}$. Which has the more, and how much?

53. How much is $\frac{1}{2}$ of $\frac{1}{2}$ of an orange? $\frac{1}{4}$ of $\frac{1}{2}$ of an orange?

54. At $\$\frac{3}{4}$ each, what will 8 books cost?

182. Written.

1. Find the cost of four pieces of cloth, measuring $23\frac{1}{2}$ yards, $25\frac{3}{4}$ yards, $22\frac{1}{8}$ yards, and $21\frac{1}{4}$ yards, at \$2 a yard.

2. Charles has $\$15\frac{3}{4}$ in the bank. How much more must he earn before he has \$50?

3. From the sum of $2\frac{1}{2}$ and $31\frac{1}{4}$, take the difference of $5\frac{7}{8}$ and $4\frac{1}{4}$. (Indicate.)

4. Which is greater, the product of $\frac{3}{4}$ and $\frac{1}{3}$, or the difference? How much?

5. A man drew from the bank \$68, which was $\frac{3}{4}$ of his entire deposit. How much had he at first? (Question.)

6. A sum of money was divided between John and James. John received $\frac{2}{5}$ of it, and James \$300. How much did John receive?

7. A merchant paid \$272 for flour at $\$\frac{5}{7}$ a barrel. How many barrels did he buy?

8. Subtract 2 from both terms of the fraction $\frac{7}{8}$. Do you increase or diminish the fraction, and how much? Add 2 to each term and find the difference.

9. What must be added to $\frac{4}{5} + 1\frac{2}{3} + \frac{1}{6}$ to make 5?

10. What number multiplied by $25\frac{1}{2}$ will make 264?

11. A lady gave $\$\frac{3}{4}$ for ribbon, $\$\frac{1}{8}$ for pins, $\$\frac{5}{6}$ for velvet, and $\$\frac{1}{6}$ for lining. What change did she receive from a five-dollar bill?

12. What will $4\frac{3}{8}$ pounds of raisins cost at 15¢ a pound?

13. From 25 yards of cloth, a tailor used $1\frac{3}{8}$ yards for making a vest, $2\frac{1}{4}$ yards for a pair of pantaloons, $3\frac{1}{2}$ yards for a coat, and $4\frac{1}{4}$ yards for an overcoat. How much cloth was left?

14. I owned $\frac{5}{4}$ of a farm and sold $\frac{1}{2}$ of my share. What part of the farm did I sell, and what part did I retain?

15. Henry had \$78 in the bank. He withdrew \$15 at one time and \$38 at another time. What part of his money did he withdraw? (Question.) What part remained? (Question.)

16. A man having 150 sheep sold $\frac{2}{3}$ of them and kept the remainder. How many were sold? How many were kept?

17. A man owning $\frac{4}{5}$ of a factory sold $\frac{2}{3}$ of his share. What part of the factory did he sell?

18. A man sold 50 sheep, which was $\frac{2}{5}$ of his whole flock. How many sheep had he at first?

19. If 4 yards of ribbon cost $35\frac{1}{2}$¢, what is the price per yard?

20. If $20\frac{3}{4}$ pounds of sugar cost 80 cents, what will 50 pounds cost? (Use cancellation.)

21. If $20\frac{3}{4}$ pounds of sugar cost 83¢, how many pounds can be bought for \$2.00?

22. I bought 55 yards of cloth at $\$\frac{5}{8}$ a yard, and sold it at $\$\frac{3}{4}$ a yard. What was the profit?

23. If an acre of land is worth \$150, what part of an acre will \$50 buy?

24. How long will it take a boy to pay for a \$75 bicycle, if he can pay $\$\frac{5}{8}$ a day?

25. $\frac{2}{5}$ of a class are boys; there are 30 girls. How many pupils in the class?

26. If $2\frac{1}{2}$ pounds of tea cost \$1.00, how much will $3\frac{3}{4}$ pounds cost?

27. At $\$\frac{1}{4}$ a pound, what is the cost of $\frac{1}{2}$ pound of tea?

28. Find the product of $\frac{2}{3}$, $\frac{6}{7}$, $5\frac{3}{4}$, and $2\frac{1}{2}$.

DECIMAL FRACTIONS

183. A Common Fraction is generally expressed by placing the denominator below the numerator.

184. A Decimal Fraction has a denominator, but the latter is not generally written.

The denominator of a common fraction may be any number.

185. The denominator of a decimal fraction *must be* 10, or 100, or 1000, etc.

NOTE. — The word *decimal* comes from the Latin word *decem*, ten, and so the denominator of every decimal fraction is 10, or 10×10, or $10 \times 10 \times 10$, etc.

186. A **Decimal Fraction**, or **Decimal**, is a fraction whose unit is divided into tenths, hundredths, thousandths, etc.

187. A Decimal is always written at the right of a period (.), called the **Decimal Point.**

We use the decimal point in writing U.S. money to separate the dollars from the cents and mills. 5 dollars and 28 cents is written \$5.28. But 28 cents is $\frac{28}{100}$ of a dollar; hundredths therefore are written like cents, with two decimal figures.

NOTE. — Any figure at the right of the decimal point is a decimal figure.

Tenths are written like dimes, with one decimal figure. $\frac{3}{10} = .3$; $\frac{8}{10} = .8$; $\frac{7}{10} = .7$.

Hundredths are written with two decimal figures, $\frac{28}{100} = .28$; $\frac{39}{100} = .39$; $\frac{6}{100} = .06$.

Thousandths are written with three decimal figures, $\frac{325}{1000} = .325$; $\frac{864}{1000} = .864$; $\frac{25}{1000} = .025$.

Name the denominators:

1. .6
2. .17
3. .105
4. .006
5. .05
6. .225

188. Change to the decimal form:

1. $\frac{25}{100}$
2. $\frac{85}{100}$
3. $\frac{326}{1000}$
4. $\frac{6}{10}$
5. $\frac{16}{100}$
6. $\frac{384}{1000}$
7. $\frac{49}{100}$
8. $\frac{5}{10}$
9. $\frac{6}{100}$
10. $\frac{15}{1000}$
11. $\frac{25}{1000}$
12. $\frac{11}{100}$
13. $\frac{9}{100}$
14. $\frac{500}{1000}$
15. $\frac{50}{100}$
16. $\frac{9}{10}$
17. $\frac{1}{10}$
18. $5\frac{40}{100}$
19. $8\frac{2}{10}$
20. $64\frac{683}{1000}$

189. Change to common fractions, and read:

21. .36
22. .7
23. .125
24. 12.2
25. 6.25
26. .485
27. .016
28. .16
29. .06
30. .6
31. 5.6
32. 5.06
33. 5.006
34. 5.600
35. 5.060

190. Write, first as common fractions, then as decimals:

36. Four *tenths*.
37. Seventy-five *hundredths*.
38. One hundred twenty-five *thousandths*.
39. Sixteen, and forty-eight *hundredths*.
40. Twelve, and four *tenths*.

41. Six *tenths.*

42. Six *hundredths.*

43. Six *thousandths.*

44. How many decimal figures are required to express thousandths? Hundredths? Tenths?

45. Read the numerators only in examples 36 to 43.

Write the following as decimals, and read the numerator of each, then the denominator of each:

46. Two hundred eighty-two thousandths.

47. Fifty-six hundredths.

48. Seven tenths.

49. Six hundred thousandths.

191. Oral.

1. What part of 10 units is 1 unit?

2. What part of 1 ten is 1 unit?

3. What part of 2 hundreds is 2 tens?

4. In the number 555, what is the value of the first 5 at the right? The second 5? The third 5?

5. Upon what does the value of any figure depend?

6. The value of the first five is what part of the value of the second five?

7. $\frac{2}{10}$ is what part of 2 units?

8. In the number 5.5, the value of the right-hand 5 is what part of the value of the left-hand 5?

9. Write five and five-tenths decimally.

10. .5 is what part of 5?

11. In the decimal .555, what is the value of the first 5 to the right? The second 5? The third 5?

12. How much greater is the value of the third 5 than the second 5?

13. How much greater is the second 5 than the first 5?

In the number 555.555, we have found that the value of each 5 is $\frac{1}{10}$ as much as the next 5 to the left. Also that the value of any 5 is 10 times as great as the value of the next 5 to the right.

In decimals as in integers, any figure removed one place to the right is diminished tenfold, and when removed one place to the left is increased tenfold.

14. In the number 32.6, what would be the value of the 2 if it were removed one place to the right? One place to the left?

Read:

15. .222

16. 2.22

17. 22.2

18. 222.

192. Integers and Decimals.

Millions.	Hundred-Thousands.	Ten-Thousands.	Thousands.	Hundreds.	Tens.	Ones.		Tenths.	Hundredths.	Thousandths.	Ten-Thousandths.	Hundred-Thousandths.	Millionths.
4,	2	4	3,	6	8	4	•	2	6	8,	5	0	4

The number is read 4 million, 243 thousand, 684 *and* 268 thousand 504 *millionths*.

193.

Rule for Reading Decimals

1. For the numerator, read the decimal as an integer.
2. For the denominator, give the place name of the last figure.

NOTE. — The denominator of a decimal is always named, but seldom written.

In reading an integer and a decimal, read "*and*" where the decimal point occurs.

What is the denominator when the decimal has one figure? Two figures? Three figures? Four figures? Five figures? Six figures? Seven figures?

194. Read:

1. .368	**7.** 28.3005	**13.** .4983695
2. .894	**8.** .28962	**14.** 4.98369
3. .5328	**9.** 15.60534	**15.** 49.8369
4. .2053	**10.** 37.00537	**16.** 498.369
5. 25.623	**11.** 25.203602	**17.** .000400
6. 7.0063	**12.** 38.000006	**18.** .0004

19. In example 11 remove the point two places to the right, and read. Four places to the right. One place to the left.

20. Read the denominators only in the last five examples.

195.

Rule for Writing Decimals

Write the numerator, prefix naughts when necessary to express the denominator, and place the point at the left.

How many decimal figures are required to express tenths? Hundredths? Thousandths? Ten-thousandths? Hundred-thousandths? Millionths? Ten-millionths?

196. Write decimally:

1. Eight *tenths*.
2. 29 *hundredths*.
3. Sixteen, and 284 *thousandths*.
4. 4584 *ten-thousandths*.
5. Twenty-five *hundredths*.
6. Twenty-five *thousandths*.
7. Twenty-five *ten-thousandths*.
8. Twenty-five *hundred-thousandths*.
9. Twenty-five *millionths*.
10. 1650, and 464 *thousandths*.
11. One thousand one, and 36 *hundred-thousandths*.
12. Sixteen, and six *thousandths*.
13. Seven hundred eighty-four *millionths*.
14. Twelve *hundred-thousandths*.
15. Seventy-five *ten-thousandths*.

197. Oral.

1. In the three decimals, .4, .40, .400, is there any difference in value?
2. What is the effect when a cipher is annexed to a decimal?
3. What is the effect when a cipher is annexed to an integer? Give an example.
4. In the decimals, .4, .04, .004, is there any difference in value?
5. What is the effect when a cipher is prefixed to a decimal? Two ciphers? Give examples.
6. What is the effect when a cipher is prefixed to an integer?

198.

Principles

1. Ciphers annexed to decimals do not change their values.

2. For each cipher prefixed to a decimal the value is diminished tenfold.

3. The denominator of a decimal when expressed is always 1 with as many ciphers as there are places in the decimal.

REDUCTION OF DECIMALS

199. Decimals may be reduced to a common denominator by annexing ciphers sufficient to give the same number of decimal figures to all the decimals.

200. Reduce to a common denominator:

1. .5, .365, and .4689.
2. .18963, .5, 7.84, .16005.
3. .28, 3.5, .00005, .256.
4. .5, .05, .005, .0005, .00005.
5. .0463, .03, .1, .100010.
6. .38, 1.16, .4, 78.592.

201. 1. Reduce .375 to a common fraction.

SOLUTION. — .375 as a common fraction is $\frac{375}{1000}$. $.375 = \frac{375}{1000} = \frac{3}{8}$. *Ans.*

Rule for reducing Decimals to Common Fractions

Write the numerator, omitting the decimal point, supply the denominator, and reduce to lowest terms.

Reduce to common fractions :

2. .25	**8.** .125	**14.** .368
3. .35	**9.** .875	**15.** 16.75
4. .75	**10.** .375	**16.** .00125
5. .64	**11.** .455	**17.** .054
6. .52	**12.** .025	**18.** .0250
7. .38	**13.** .561	**19.** .01375

20. Reduce $.37\frac{1}{2}$ to a common fraction.

SOLUTION. — $.37\frac{1}{2} = \frac{37\frac{1}{2}}{100} = \frac{\frac{75}{2}}{100} = \frac{75}{200} = \frac{3}{8}$.

202. Reduce to common fractions :

21. $.12\frac{1}{2}$	**24.** $.18\frac{1}{8}$	**27.** $.87\frac{1}{2}$
22. $.62\frac{1}{2}$	**25.** $.03\frac{3}{4}$	**28.** $.66\frac{2}{3}$
23. $.06\frac{1}{4}$	**26.** $.25\frac{4}{5}$	**29.** $.36\frac{7}{8}$

Reduce to mixed numbers :

30. 16.25, $2.33\frac{1}{3}$, 34.75.

203. Reduce $\frac{3}{4}$ to a decimal.

$\frac{3}{4}$ = 3 times $\frac{1}{4}$. 3 = (3.0) 30 tenths.
$\frac{1}{4}$ of 3.0 = (.7) 7 tenths and 2 tenths remainder.
2 tenths = 20 hundredths. $\frac{1}{4}$ of .20 = .05.
Hence $\frac{3}{4}$ = .7 + .05 = .75.

Rule for reducing Common Fractions to Decimals

Annex decimal ciphers to the numerator, and divide by the denominator. Point off in the quotient as many decimal places as there are ciphers annexed.

The division will not always be exact. In such cases, write the remainder over the divisor as a common fraction, or place the sign (+) after the decimal to show that the result is incomplete.

Thus, $\frac{1}{7} = .142\frac{6}{7}$, or .142 +.

204. Reduce to decimals:

1. $\frac{4}{5}$	**5.** $\frac{3}{20}$	**9.** $\frac{7}{15}$	**13.** $\frac{1}{8}$	**17.** $\frac{3}{80}$
2. $\frac{3}{4}$	**6.** $\frac{2}{5}$	**10.** $\frac{54}{36}$	**14.** $\frac{56}{25}$	**18.** $\frac{8}{25}$
3. $\frac{4}{9}$	**7.** $\frac{3}{5}$	**11.** $\frac{12}{40}$	**15.** $3\frac{1}{16}$	**19.** $\frac{15}{20}$
4. $\frac{7}{8}$	**8.** $\frac{5}{9}$	**12.** $\frac{9}{16}$	**16.** $\frac{3}{8}$	**20.** $2\frac{3}{32}$

205. ADDITION

Add: .35, 4.375, 28.3065.

.35
4.375
28.3065

Rule for Addition of Decimals

Write the numbers so that the decimal points stand in a column. Add as in integers, and place the point in the sum directly under the points above.

Add: **2.**

3.25
7.163
15.0032

3.

4.5
.168
2.12

4.

.004
4.1
16.1563

5. .175 + 1.75 + 17.5 + 175.

6. 145 + 14.5 + 1.45 + .145 + .0145.

7. 3.2 + 14.0063 + .006 + 25.384 + .1.

8. $.8 + .446 + 59.3 + 2.575 + 1.0056 + .3$.

9. $1.45 + 2.365 + 96 + .96 + 15.863 + 4.3 + .0004$.

10. $446 + 44.6 + 37562 + 9 + .8 + .321 + .16$.

11. $21.0005 + .3842 + .1 + .005 + 3.6 + .158$.

12. $1.0006 + 2001.1 + .003 + 5.5 + 11.1111$.

206. SUBTRACTION

Rule for Subtraction of Decimals

Write the numbers so that the decimal point of the subtrahend stands directly under the decimal point of the minuend, subtract as in integers, and place the point directly under the points above.

NOTE. — It is sometimes convenient to give the decimals the same denominator, by annexing decimal ciphers.

Subtract:

1.	2.	3.	4.
24.3	2.86	4.	2.46
4.5	1.325	1.15	.005

Find the remainders:

5. $7 - .15$
6. $1 - .004$
7. $13 - 2.1$
8. $3.256 - 1.05$
9. $256.1 - 1.256$
10. $29.325 - 15.14$
11. $3.852 - .125$
12. $1.1111 - .0011$
13. $500 - .05$
14. $25.3894 - 15.005$

15. From twenty-eight, and twenty-five thousandths take fourteen, and twenty-five hundredths.

16. From one tenth take one thousandth.

17. Which is the greater, fifty thousandths or five hundredths?

18. Take one thousandth from one thousand.

19. From 5 hundred take 5 hundredths.

MULTIPLICATION

207. Oral.

How much is 2 times .3? 3 times .3? 4 times .3?

7 times .02 = ? 12 times .06 = ? 12 times $.12 = ?

$\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$; $1 \times .1 = .01$.

$\frac{3}{10} \times \frac{5}{100} = \frac{15}{1000}$ $.3 \times .05 = .015$.

$\frac{3}{10} \times \frac{3}{10} = \frac{9}{100}$; $.3 \times .3 = .09$.

$\frac{5}{100} \times \frac{3}{10} = \frac{15}{1000}$ How many ciphers in the denominator of the product?

How many ciphers in the denominators of both factors?

Every decimal has a corresponding common fraction, and for each cipher in the denominator of the common fraction there is a decimal figure in the decimal.

$.05 \times .3 = .015$ How many decimal figures in both factors?

Rule for Multiplication of Decimals

Multiply as in integers, and point off from the right of the product as many decimal figures as there are decimal figures in both factors.

NOTE. — If there are not figures enough, prefix ciphers.

Ciphers at the right of a decimal have no value, and may be omitted.

2.8	1.25	.005	25
× 8	.6	.03	.06
22.4	.750	.00015	1.50

Find the products:

1. $.18 \times .15$
2. $1.0005 \times .2$
3. $2.5 \times .06$
4. $56 \times .005$
5. $.005 \times 1.6$
6. 25.05×1.15
7. 2.863×100
8. 13.3×1.3
9. $100 \times .01$
10. $100.56 \times .0005$
11. 25.32×1.05
12. $2.84 \times 2\frac{1}{2}$
13. $3.28 \times 12\frac{1}{2}$
14. 1.111×1000

208. 1. Multiply 1.265 by 100.

$$\begin{array}{r} 1.265 \\ 100 \\ \hline 126.500 \end{array}$$

To Multiply by 10, 100, 1000, etc.

Remove the decimal point one place to the right for every naught in the multiplier.

Do not write the multiplier.

Oral. Multiply:

2. 3.84 by 10
3. 12.63 by 100
4. 1.5555 by 1000
5. 1.358 by 10
6. .25 by 1000
7. .3 by 100
8. 1.869 by 100
9. 32.856 by 1000
10. 138.56 by 1000
11. 11.11 by 100

DIVISION

209. Since in multiplication there are as many decimal figures in the product as in both factors, in division the quotient will have as many decimal figures as the number of decimal figures in the dividend exceeds those in the divisor.

$$.5\overline{)\,12.685}\quad 25.37$$

Since there are three decimal figures in the dividend and one in the divisor, there must be two in the quotient. Prove by multiplying dividend by quotient.

Divide 399.552 by 192.

$$\begin{array}{r} 2.081 \\ 192\overline{)\,399.552} \\ \underline{384} \\ 1555 \\ \underline{1536} \\ 192 \\ \underline{192} \end{array}$$

Rule for Division of Decimals

In all cases divide as in integers, then place the decimal point.

When the divisor is an integer, place the point in the quotient directly over the point in the dividend, in long division (directly under in short division).

Prove by multiplying the divisor by the quotient.

When the divisor contains decimal figures, move the point in both divisor and dividend as many places to the right as there are decimal figures in the divisor, then place the point in the quotient as if the divisor were an integer.

NOTE 1. — The new point in both dividend and divisor may be placed on a line with the tops of the figures, and the original point may stand, to preserve the reading of the decimals.

NOTE 2. — In the above example, the moving of the point two places to the right in both dividend and divisor is equivalent to multiplying each by 100.

NOTE 3. — If the quotient does not have a sufficient number of figures, prefix ciphers.

NOTE 4. — Before commencing to divide see that there are at least as many decimal places in the dividend as in the divisor.

NOTE 5. — If there is a remainder, after all the figures of the dividend are used, annex decimal ciphers and continue the division.

NOTE 6. — It is not usually necessary to have more than four decimal figures in the quotient.

Divide 28.78884 by 1.25.

$$
\begin{array}{r}
23.031+ \\
1.25\overline{)28.78884} \\
250 \\
\hline
378 \\
375 \\
\hline
388 \\
375 \\
\hline
134 \\
125 \\
\hline
9
\end{array}
$$

Divide .125 by .5.
Divide 1.25 by .5.
Divide 12.5 by .5.

Divide at sight:

1. $3\overline{)3.33}$	4. $.03\overline{).333}$	7. $.5\overline{)25}$
2. $3\overline{).333}$	5. $.003\overline{).333}$	8. $.05\overline{).25}$
3. $.03\overline{)333}$	6. $5\overline{)2.5}$	9. $.07\overline{).28}$

Find quotients and prove:

210.

10.	$3.57 \div .7$	**18.**	$376 \div .6$
11.	$.488 \div .12$	**19.**	$376 \div .06$
12.	$16.55 \div .05$	**20.**	$37.6 \div .6$
13.	$13.13 \div 1.3$	**21.**	$3.76 \div .06$
14.	$1.111 \div .01$	**22.**	$1.875 \div .005$
15.	$5555 \div .5$	**23.**	$15.55 \div .1$
16.	$.875 \div .05$	**24.**	$1 \div .1$
17.	$73.5 \div 1.05$	**25.**	$.1 \div 1$

Divide:

26. Twenty-five ten-thousandths by 25 hundredths.

27. 4678 hundred-thousandths by 9 ten-thousandths.

28. 3582 ten-thousandths by 3 hundredths.

To divide by 10, 100, 1000, etc.

Remove the decimal point one place to the left for each cipher in the divisor.

Divide at sight:

29.	$10\overline{)365.8}$	**33.**	$100\overline{)189.36}$	**37.**	$1000\overline{)1698.45}$
30.	$10\overline{)5}$	**34.**	$100\overline{).189}$	**38.**	$1000\overline{)1.111}$
31.	$10\overline{)148.963}$	**35.**	$100\overline{)148.369}$	**39.**	$1000\overline{)2948.36}$
32.	$10\overline{)115.55}$	**36.**	$100\overline{)4.983}$	**40.**	$1000\overline{)39.85}$

Read and add:

1.

32.065
6.006
4.25
.032
.25
25.01
111.11

2.

36.9486
2583.04
4.9602
15.15
100.001
56.56
141.141

REVIEW OF DECIMALS

211. **1.** Tell where to place the decimal point in any product.

2. In any quotient.

3. In any remainder.

4. In any sum.

5. From 1 take 1 millionth.

6. Add 1 tenth, 1 hundredth, and 1 thousandth.

7. Find the product of 1 multiplied by .15.

8. Multiply at sight: 36.984 by 1000.

9. Divide at sight: 159.83 by 1000.

Change to decimals:

10. $\frac{7}{8}$, $\frac{5}{7}$, $\frac{4}{5}$, $16\frac{3}{4}$, $25\frac{7}{8}$.

Change to common fractions:

11. .28, .38, .375, 15.125.

12. If John earns \$8 in a week, how much can he earn in 7.5 weeks?

13. If a barrel of flour costs \$5.25, how many barrels can be bought for \$105?

14. What is the effect when a decimal figure is removed one place to the left? To the right?

15. What is the effect when an integral figure is removed one place to the left? One place to the right?

16. At 15¢ a peck, how many pecks of pop-corn can be bought for \$3.75?

17. What is the cost of 28.78 yards of cloth at \$3.15 a yard?

BILLS AND ACCOUNTS

212. An **Account** is a record of indebtedness for articles bought or sold, cash paid or received, or services rendered.

213. A **Debtor** is a person who owes a debt.

214. A **Creditor** is a person to whom a debt is owed.

215. A **Bill** is a written statement of a debtor's account, made by the creditor.

216. A **Receipt** is a creditor's written acknowledgment that he has received payment of part or all of a debt.

217. A bill is **Receipted** when its payment is acknowledged in writing, by the creditor, or by some authorized person.

NOTE. — The sign @ is for at, Dr. is for debtor, and Cr. for creditor.

1. BILL FORMS

DETROIT, MICH., *July 1, 1901.*

JAMES P. BARNES,

Bought of DEY BROS. & Co.

	50 yards Brussels Carpet	@	$1 15	$
	24 " Oilcloth	"	35	
	4 dozen pair Merino Hose	"	3 50	
	2 Willow Chairs	"	4 50	
				$

2. FORM OF A RECEIPTED BILL

NEW YORK, *June 30, 1902.*

JEROME A. PHELPS,

In account with D. O. POTTER & Co.

					$	
May	*14*	*12 barrels Flour*	@ $	*6.50*		
"	*14*	*6 tubs Butter, 684 pounds*	"	*.24*		
June	*10*	*5 barrels beef*	"	*25.28*		
"	*25*	*450 pounds Ham*	"	*.09¼*		

Received payment,

D. O. POTTER & Co.

3. Mr. John Q. Adams buys of D. McCarthy & Co.:

4 pounds of coffee at 27 cents a pound.
18 pounds of sugar at 5½ cents a pound.
5 gallons of molasses at 60 cents a gallon.
16 pounds of rice at 8¼ cents a pound.

Make out the bill.

4. James Smith, farmer, sold Richard Dunn, grocer, the following: 6 barrels of potatoes at $1.80 a barrel.
2 tons of hay at $16 a ton.
3 cords of wood at $4 a cord.
360 pounds of butter at 24½¢ a pound.

Make a receipted bill.

5. Syracuse, Dec. 5, 1898. Edward Smith sold B. M. Watson 65 yards of Brussels carpet at $1.25; 24 yards of oil-cloth @ 35¢; one dozen pair of merino hose @ $3.50; 2 willow chairs @ $4.50. Make a bill, find the footing, and properly receipt it.

6. Make out a bill of groceries. Foot it, and receipt it, with F. H. Mead as creditor and Wm. H. Scott as debtor.

TO THE TEACHER. — See that the prevailing prices are used, and that the quantities are consistent.

DENOMINATE NUMBERS

218. A number composed of units which belong to a table of weights or measures is a **Denominate Number.** Thus, 2 feet, 7 gallons, 3 hours, are denominate numbers.

219. A number composed of two or more kinds of units belonging to the same table is a **Compound Number.** Thus, 2 yards, 1 foot, 6 inches, and 1 ton, 50 pounds, 11 ounces, are compound numbers.

LINEAR MEASURES

220. Measures used in measuring distances and dimensions are **Linear Measures.**

221. The **Yard** is the standard unit of linear measure.

With a yardstick draw a line one yard long. Hold your hands one yard apart. Name objects one yard apart. How many yards apart are the windows of your schoolroom? Without the measure, draw a line one yard long. Correct it. Divide it into 3 equal parts. Each part is one foot long.

Hold your hands one foot apart. Measure a foot on your arm. Name objects 1 foot long, wide, or high.

Draw a line one foot long. Correct it with a rule. Divide it into 12 equal parts. Each part is 1 inch. Show how long 1 inch is. Four inches. Six inches.

With a yardstick, measure $5\frac{1}{2}$ yards on the schoolroom floor. This is 1 rod. How many feet make a rod? How many rods long is your school ground? (Estimate it.) How wide? Where would you stop if you should walk 20 rods from the front door of your schoolhouse?

320 rods make 1 mile. Name some places 1 mile from your school. How many miles do you walk in coming to school? How many yards in 1 mile? How many feet? How many inches?

222. The answers to the above questions give us the following

TABLE OF LINEAR MEASURES

12	inches (in.)	make 1	foot	(ft.).
3	feet	make 1	yard	(yd.).
$5\frac{1}{2}$	yards	make 1	rod	(rd.).
320	rods	make 1	mile	(mi.).
5280	feet	make 1	mile.	

Oral.

1. How many inches are there in 1 yd.?
2. How many inches are there in 2 ft.? 3 ft.? 5 ft.?
3. How many feet are there in 4 rd.? 2 rd.?
4. How many feet are there in 5 yd.? 7 yd.? 11 yd.?
5. 24 feet are how many yards?
6. How many feet around a picture 2 ft. long and 1 ft. wide?

7. How many inches in a half yard?

8. At \$.16 a yard, what will $1\frac{1}{2}$ yd. of ribbon cost?

9. 6 ft. + 2 ft. + 4 ft. are how many yards?

10. How many feet in 108 inches?

11. John has a fish polé 4 yd. long. How many inches long is it?

12. How many rods in 5 miles? 7 miles? $1\frac{1}{2}$ miles?

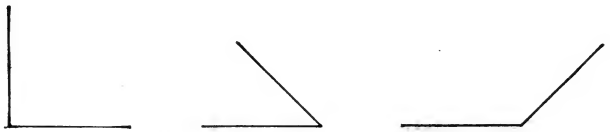
13. How many feet in $\frac{1}{2}$ mile? 3 miles? $1\frac{1}{2}$ miles?

14. Stepping two feet at a step, how many steps will be taken in walking 2 miles?

SURFACE MEASURES

223. A **Surface** is that which has only length and breadth. Thus, the top of a desk, the outside of a book, the upper and under sides of a board, are surfaces.

224. An **Angle** is the difference in direction of two lines that meet. Thus,

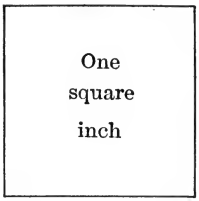


225. A **Plane Surface** is a surface which would be touched by all the points of a straight line drawn in any direction upon it. Thus, the top of a table is a plane surface.

226. A **Plane Figure** is a portion of a plane surface bounded by lines. Thus, a triangle, an oblong, a circle, are plane figures.

227. A **Square** is a plane figure bounded by four equal straight sides and having four equal angles.

A square whose side is one inch is a **Square Inch.** Thus,



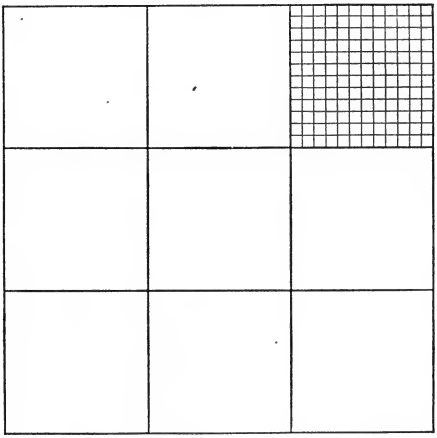
A square whose side is one foot is a **Square Foot.** Draw a square foot.

A square whose side is one yard is a **Square Yard.** Draw a square yard.

A square whose side is one rod is a **Square Rod.** Measure a square rod in your schoolroom.

A square whose side is one mile is a **Square Mile.** A **Section** of land is a square mile.

Draw a square yard on the blackboard. Divide each side into feet and connect the division marks as in the figure. How many squares are there? What is each square? How many square feet are there in one square yard?



NOTE.—Be sure to make the figure on the blackboard *full size.*

Divide the sides of one of these square feet into twelve equal parts. Connect the division marks.

How many squares have you made? What is each square?

How many square inches make one square foot?

In a similar manner you might divide the sides of a square mile into 320 rods each, making $320 \times 320 = 102400$ square

rods in one square mile. Each square mile, however, is divided into 640 acres. This makes how many square rods in one acre?

You might also divide a square rod into $5\frac{1}{2} \times 5\frac{1}{2} = 30\frac{1}{4}$ square yards.

228. The foregoing demonstration gives us the following

TABLE OF SURFACE MEASURES

144 square inches (sq. in.) make 1 square foot (sq. ft.).
9 square feet make 1 square yard (sq. yd.).
$30\frac{1}{4}$ square yards make 1 square rod (sq. rd.).
160 square rods make 1 acre (A.).
640 acres make 1 square mile (sq. mi.).

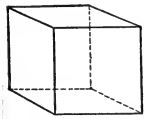
229. Written.

1. How many square inches in 3 sq. feet? in 6 sq. ft.?
2. How many square yards in 54 sq. feet? in 108 sq. ft.?
3. In $12\frac{1}{2}$ sq. ft. how many sq. inches?
4. How many acres in 480 square rods? in 640 sq. rd.?
5. In 7212 sq. in., how many sq. feet?
6. In 40 acres, how many square rods?
7. In $\frac{1}{4}$ of an acre, how many sq. rods?
8. A farmer had a section of land. He sold $\frac{1}{2}$ of it to one man and $\frac{1}{4}$ to another. How many acres had he left? What part of the farm is left?
9. 3 square feet is what part of a square yard?
10. What part of a square foot is 36 square inches? 108 sq. in.?
11. What part of an acre is 120 square rods?

CUBIC MEASURES

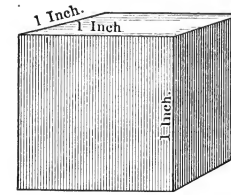
230. That which has length breadth, and thickness is a **Solid.**

231. A solid which has six equal square faces is a **Cube.**



Cube

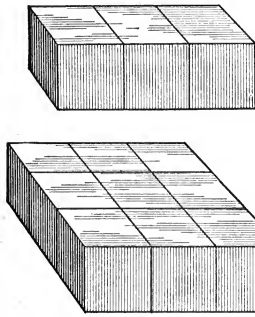
A cube whose edge is one inch is a **Cubic Inch.**

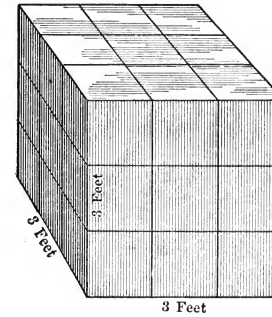


Cubic inch

A cube whose edge is one foot is a **Cubic Foot.**

A cube whose edge is one yard is a **Cubic Yard.**





A solid 3 feet long, 1 foot wide, and 1 foot high contains how many cubic feet? $(3 \times 1 \times 1.)$ (See picture.)

A solid 3 feet long, 3 feet wide, and 1 foot high contains how many cubic feet? $(3 \times 3 \times 1.)$ (See picture.)

A solid 3 feet long, 3 feet wide, and 3 feet high contains how many cubic feet? (See picture.)

A cubic yard is how many feet long, wide, and high?

Therefore, a cubic yard contains how many cubic feet?

A solid 12 inches long, 1 inch wide, and 1 inch high contains how many cubic inches?

A solid 12 inches long, 12 inches wide, and 1 inch high contains how many cubic inches?

A solid 12 inches long, 12 inches wide, and 12 inches high contains how many cubic inches? How many cubic feet?

Therefore, a cubic foot contains how many cubic inches?

232. The answers to the above questions give us the following

TABLE OF CUBIC MEASURES

1728 cubic inches (cu. in.) make 1 cubic foot (cu. ft.).
27 cubic feet make 1 cubic yard (cu. yd.).

Written.

1. How many cubic inches in 6 cubic feet? in 8 cu. ft.?

2. How many cubic feet in 8640 cubic inches? in 11,232 cu. in.?

3. How many cubic inches in $\frac{1}{8}$ of a cubic foot? in $\frac{1}{2}$ cu. ft.?

4. What part of a cubic foot is 576 cubic inches? 1152 cu. in.?

5. What part of a cubic yard is 3 cubic feet? 9 cubic feet?

6. What will it cost to remove an embankment containing 54,000 cu. ft. of earth at $12\frac{1}{2}$ cents a cubic yard?

LIQUID MEASURES

1 Gill 1 Pint 1 Quart 1 Gallon

233. Fill a gill cup with water. Pour it into a pint cup. Repeat until the pint cup is full. How many times have you filled the gill cup? How many gills in one pint? Fill a pint cup and pour into a quart cup until the quart cup is full. How many pints in one quart? Fill a gallon measure with the quart cup. How many quarts in one gallon? If you should fill a barrel with the gallon measure, you would need to fill the gallon measure $31\frac{1}{2}$ times. How many gallons make one barrel? Two barrels of water would fill a hogshead. How many gallons in a hogshead?

234. From this work we may make the following

TABLE OF LIQUID MEASURES

4	gills (gi.)	make 1 pint (pt.).
2	pints	make 1 quart (qt.).
4	quarts	make 1 gallon (gal.).
$31\frac{1}{2}$	gallons	make 1 barrel (bbl.).
2	barrels	make 1 hogshead (hhd.).

These denominations are sometimes used:

2 hhd. = 1 pipe (pi.).
2 pi. = 1 tun.

Oral.

1. How many pint cups can be filled from 8 quarts?

2. A quart of milk was taken from a five-gallon pan. How much was left?

3. How many gallons in a hogshead?

4. $\frac{3}{4}$ of a gallon are how many quarts?

5. How many hogsheads will 8 barrels of oil fill?

DRY MEASURES

235. Fill a pint measure, which is used for dry measures, and empty it into a quart measure, continuing until the latter is full. How many times must you fill the pint measure? How many pints in 1 quart? Fill the quart (dry) measure and empty into a peck measure until the latter is filled. How many quarts in 1 peck? In the same way find how many pecks in 1 bushel. (Sawdust or oats may be conveniently used for this measurement.)

236. From this work we may make the following:

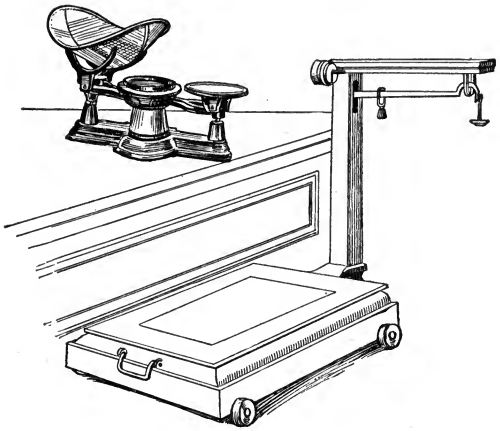
TABLE OF DRY MEASURES

2 pints (pt.)	make 1 quart (qt.).
8 quarts	make 1 peck (pk.).
4 pecks	make 1 bushel (bu.).

Oral.

1. How many pecks in 9 bushels?
2. In 22 pints, how many quarts?
3. A bushel of apples at $.25 a peck will cost what?
4. How many pints are in 6 qt. 1 pt.?
5. What will a peck of chestnuts bring if sold at 10¢ a quart?
6. How many times can a quart cup be filled from a bushel of walnuts?
7. How many quarts are in 9 pecks?

AVOIRDUPOIS WEIGHT



237. Avoirdupois weight is used in weighing all common articles, as coal, groceries, hay, etc.

TABLE OF AVOIRDUPOIS WEIGHT

16 drams (dr.)	make	1 ounce (oz.).
16 ounces	make	1 pound (lb.).
25 pounds	make	1 quarter (qr.).
4 quarters	make	1 hundred-weight (cwt.).
20 hundred-weight	make	1 ton (T.).
2000 pounds	make	1 ton.

The avoirdupois pound contains 7000 grains.
The hundred-weight is sometimes called a cental.

Oral.

1. How many ounces are in 4 pounds?

2. In 32 drams, how many ounces?

3. What will a pound of candy cost at $.02 an ounce?

4. How many pounds in 4 tons?

5. In 6000 lb., how many tons?

6. How many pounds are in a hundred-weight?

7. Paid $.48 a pound for candy. How much is that an ounce?

8. How many 3-pound packages can be made from 75 pounds of coffee?

Written.

1. How many ounces are there in 25 lb.?

2. In $7\frac{1}{2}$ tons, how many pounds?

3. How many bullets weighing 2 oz. each can be made from 32 lb. 4 oz. of lead?

4. What will $2\frac{1}{2}$ lb. dried peaches cost at $.32 per pound?

5. In 1364 drams, how many ounces?

6. Five boys share equally 1 lb. 9 oz. of candy. How many ounces do each receive?

7. How many 4-ounce bags can I fill from a box holding $7\frac{1}{2}$ lb. of candy?

8. What is the difference in ounces between a hundred-weight and $97\frac{1}{4}$ pounds?

9. What will $7\frac{3}{4}$ lb. of indigo cost at $.03 an ounce?

10. Reduce $18\frac{1}{4}$ lb. to ounces.

11. A hundred-weight is what part of a ton?

TROY WEIGHT

238. Troy weight is used for weighing gold, silver, and precious stones.

TABLE OF TROY WEIGHT

24 grains (gr.) make 1 pennyweight (pwt.).
20 pennyweights make 1 ounce (oz.).
12 ounces make 1 pound (lb.).

Can you tell why Troy weight has no larger denomination than pounds?

Oral.

1. How many ounces in 2 lb.? 5 lb.? 10 lb.?
2. How many ounces in 100 pwt.? 200 pwt.? 80 pwt.?
3. How many pwt. in 96 grains? 72 gr.?
4. How many gr. in 1 pwt.? in 2 pwt.? in 10 pwt.?
5. How many lb. in 96 oz.? in 120 oz.?
6. How many grains in half an ounce of gold?
7. A watch chain weighs 10 pwt. What part of an ounce does it weigh? How many grains?

APOTHECARIES' WEIGHT

239. Apothecaries' weight is used by physicians and druggists in compounding medicines, and by druggists in selling medicines in quantities smaller than one ounce. Medicines in quantities of one ounce and more are bought and sold by Avoirdupois weight.

TABLE OF APOTHECARIES' WEIGHT

20 grains (gr.)	make 1 scruple (sc. or ℈).
3 scruples	make 1 dram (dr. or ʒ).
8 drams	make 1 ounce (oz. or ℥).
12 ounces	make 1 pound (lb. or ℔).

Oral.

1. How many grains in 1 dram? in 1 ounce?
2. How many drams in 1 lb.? in 30 scruples?
3. How many pounds in 144 ounces? in 292 drams?
4. An ounce of quinine will make how many 4-grain powders?
5. How many drams will make sixty 3-grain tablets?

240. FEDERAL MONEY

TABLE OF FEDERAL MONEY

10 mills (m.)	make 1 cent (ct.).
10 cents	make 1 dime (di.).
10 dimes	make 1 dollar ($).
10 dollars	make 1 eagle (E.).

Oral.

1. How many dimes in 6 dollars?
2. 4 eagles are how many dollars?
3. A man had $80 in eagles. How many eagles had he?
4. How many mills in 15 cents?
5. In 5 dimes, how many cents?
6. A dollar, a quarter, a dime, and a nickel are how many cents?
7. How many dollars are there in 130 dimes?
8. How many dollars are there in 1300 cents?

9. Divide 4 eagles among 5 men. How much will each receive?

10. How many books at 50¢ each can be bought for $2?

11. How many dollars in a double eagle?

TIME

21. The solar year is $365\frac{1}{4}$ days, nearly.

For convenience 365 days is taken for a common year. The $\frac{1}{4}$ day, in 4 years, amounts to another day, making every 4th year 366 days. This is called leap year. This extra day is added to February.

Days in each month:

January 31 days.
February 28 or 29 days.
March 31 days.
April 30 days.
May 31 days.
June 30 days.
July 31 days.
August 31 days.
September 30 days.
October 31 days.
November 30 days.
December 31 days.

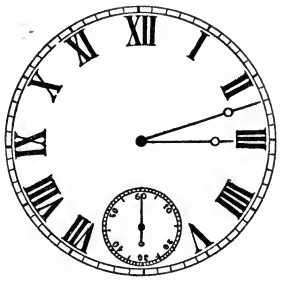


TABLE OF TIME

60 seconds (sec.)	make 1 minute (min.).
60 minutes	make 1 hour (h.).
24 hours	make 1 day (da.).
365 days	make 1 common year (y.).
366 days	make 1 leap year.

Also:

7 days	make 1 week (wk.).
12 months (mo.)	make 1 year.
100 years	make 1 century.

Oral.

1. How many minutes in 2 hours? $\frac{1}{2}$ hour? $2\frac{1}{2}$ hours?

2. Reduce to hours: 180 minutes, 120 minutes, 30 minutes.

3. How many hours in a week?

4. How many days in March and June?

5. What 4 months have 30 days?

6. How many have all the rest except February?

7. What part of an hour is 45 minutes? (Question.)

Written.

1. If a boy walk $\frac{1}{2}$ mile in ten minutes, how long will it take him to walk 16 miles?

2. How many hours in the month of August?

The winter months are December, January, and February.

The spring months are March, April, and May.

The summer months are June, July, and August.

The autumn months are September, October, November.

3. Which is the shortest season? Why?

4. Is it the shortest in leap year?

5. How many more days in this year?

MISCELLANEOUS

12 units or things, 1 dozen (doz.).
12 dozen, 1 gross (gro.).
20 units, 1 score.
196 lb., 1 barrel of flour.
200 lb., 1 bbl. of beef, pork, or fish.
280 lb., 1 barrel of salt.

TABLE OF PAPER MEASURE

24 sheets	make 1 quire.
20 quires	make 1 ream.
2 reams	make 1 bundle.
5 bundles	make 1 bale.

242. Oral.

1. How many are 3 gross of pencils?
2. What is the cost of $\frac{1}{2}$ gross of pens at 60¢ a gross?
3. How many are 3 score years?
4. How many are 3 score and 10 years?
5. How many are 5 dozen eggs? $6\frac{1}{2}$ dozen eggs?
6. How many sheets in 3 quires of paper?
7. How many quires in $\frac{1}{2}$ a ream?
8. 84 eggs are how many dozen?
9. How many reams in a bale of paper?
10. How many pounds in $\frac{1}{4}$ bbl. of flour?

Written.

11. How many dozen in 66 cucumbers?
12. What is the cost of 6 quires of paper at $4 a ream?
13. What will 8 gross of copy-books cost at 10¢ apiece?
14. If paper is bought at $2 a ream and sold at 18¢ a quire, what is the gain on 6 reams?

REDUCTION OF DENOMINATE NUMBERS

243. Changing numbers to smaller denominations is **Reduction Descending.**

Changing numbers to larger denominations is **Reduction Ascending.**

What is reduction (see definition, Art. 129)? In reduction of numbers, what is changed? What is not changed?

1. Reduce 2 lb. 7 oz. 5 pwt. 17 gr. to grains.

2 (lb.)	
12 oz.	
24 oz.	
+ 7 oz.	How many oz. are there in 1 lb.?
31 (oz.)	How many oz. are there in 2 lb.?
20 pwt.	24 oz. + 7 oz. = how many oz.?
620 pwt.	How many pwt. are there in 1 oz.?
+ 5 pwt.	How many pwt. are there in 31 oz.?
625 (pwt.)	620 pwt. + 5 pwt. = how many pwt.?
24 gr.	How many gr. are there in 1 pwt.?
2500	How many gr. are there in 625 pwt.?
1250	15000 gr. + 17 gr. = how many gr.?
15000 gr.	
+ 17 gr.	
15017 gr. *Ans.*	

2. Reduce 2 mi. 51 rd. 2 yd. 2 ft. 7 in. to inches.

2 (mi.)	
320 rd.	
640 rd.	
+ 51 rd.	
2|691 (rd.)	
$5\frac{1}{2}$	
$345\frac{1}{2}$	How many rods are there in 1 mi. In 2 mi.?
3455	640 rd. + 51 rd. = how many rods?
$3800\frac{1}{2}$ yd.	How many yd. in 1 rod? In 691 rd.?
+ 2 yd.	$3800\frac{1}{2}$ yd. + 2 yd. = how many yd.?
$3802\frac{1}{2}$ (yd.)	How many feet in 1 yard? In $3802\frac{1}{2}$ yards?
3	$11407\frac{1}{2}$ ft. + 2 ft. = how many feet?
$11407\frac{1}{2}$ ft.	In 1 foot there are how many inches? In $11409\frac{1}{2}$
+ 2 ft.	feet?
$11409\frac{1}{2}$ (ft.)	136914 in. + 7 in. = how many inches?
12	
6 in.	
136908 in.	
136914 in.	
+ 7 in.	
136921 in. *Ans.*	

244. From the above examples we may make the following

Rule for Reduction Descending

Multiply the number of the largest denomination given by the number of units of the next smaller denomination which are equal to one unit of the denomination multiplied. To this product, add the given number of the same denomination as the product. Proceed in the same way with this and each succeeding result until the required denomination is reached.

3. Reduce 2 sq. mi. 125 A. 71. sq. rd. 1 sq. yd. to square yards.

2 (sq. mi.)
640 A.
1280 A.
+ 125 A.
1405 (A.)
160 sq. rd.
84300
1405
224800 sq. rd.
+ 71 sq. rd.
224871 (sq. rd.)
121 fourths sq. yd.
224871
449742
224871
4 fourths | 27209391 fourths sq. yd.
6802347¾ sq. yd. *Ans.*

NOTE. — $30\frac{1}{4}$ sq. yd. = $\frac{121}{4}$ sq. rd.

245.

Reduce to lower denominations :

4. 17 yd. 2 ft. 9 in. to inches.

5. 46 rd. 4 yd. 2 ft. to feet.

6. 3 mi. 75 rd. 4 ft. to inches.

7. 16 A. 140 sq. rd. 26 sq. yd. to square yards.

8. 4 A. 15 sq. rd. 4 sq. ft. to square inches.

9. 16 cu. yd. 25 cu. ft. 900 cu. in. to cubic inches.

10. 15 gal. 3 qt. 1 pt. to pints.

11. 7 bu. 3 pk. 5 qt. 1 pt. to pints.

12. $16\frac{1}{2}$ bu. to quarts.

13. 25 lb. 5 oz. 16 pwt. 10 gr. to grains.

14. 2 T. 6 cwt. 10 lb. 14 oz. to ounces.

15. What will 3 reams of paper cost at 40 ¢ a quire?

16. Reduce 3 mi. 4 fur. 20 rd. 5 yd. 2 ft. 8 in. to inches.

17. Reduce 6 mi. 240 rd. to feet.

18. Reduce 3 A. 8 sq. rd. 5 sq. yd. 3 sq. ft. to square inches.

19. Reduce 16 cu. yd. 9 cu. ft. 3 cu. in. to cubic inches.

20. Reduce 2 T. 3 ctl. 16 lb. to ounces.

21. Reduce 3 lb. 9 oz. 15 pwt. 12 gr. to grains.

22. Reduce 60 gal. 3 qt. 3 gi. to gills.

23. How many sheets in 5 bales of paper?

24. Reduce 3 wk. 6 da. 5 hr. to minutes.

REDUCTION ASCENDING

246. 1. Reduce 5499 qt. to bushels.

		How many qt. in 1 pk.?
8 qt.	5499 qt.	5499 qt. = how many pk.?
4 pk.	687 pk. + 3 qt.	How many qt. over?
	171 bu. + 3 pk.	How many pk. in 1 bu.?
171 bu. 3 pk. 3 qt. *Ans.*		687 pk. = how many bu.?
		How many pk. over?

2. Reduce 241329 in. to larger denominations.

12 in. | 241329 in.
3 ft. | 20110 ft. + 9 in.
| 6703 yd. + 1 ft.
| 2
11 half yd. | 13406 half yd.
| 1218 rd. + 8 half yd. = 4 yd.

3 mi. + 258 rd.
320 rd.)1218 rd.
96
258 rd.

3 mi. 258 rd. 4 yd. 1 ft. 9 in. *Ans.*

How many inches make 1 ft.?
How many feet in 241329 in.?
How many inches left?
How many ft. make 1 yd.?
How many yd. in 6703 ft.?
How many feet left?
How many yd. in 1 rd.?
How many half yd.?
How many half yd. in 6703 yd.?
How many rd. in 13406 half yd.?
How many half yd. over?
8 half yd. = how many yd.?
How many rd. in 1 mi.?
How many mi. in 1218 rd.?
How many rd. left?

3. Reduce 208824 sq. in. to larger denominations.

1450 sq. ft.
144)208824
144
648
576
722
720
24 sq. in.

$\frac{3}{4}$ sq. yd. = 6 sq. ft. 108 sq. in.
5 sq. rd. $9\frac{3}{4}$ sq. yd. 1 sq. ft. 24 sq. in. =
5 sq. rd. 9 sq. yd. 7 sq. ft. 132 sq. in. *Ans.*

9 | 1450 sq. ft.
161 sq. yd. + 1 sq. ft.
4
121 fourths)644 fourths sq. yd.

5 sq. yd. + $\frac{32}{4}$ sq. yd. = $9\frac{3}{4}$ sq. yd.

Rule for Reduction Ascending

Divide the given number by the number of units of the denomination given which are equal to one unit of the denomination next larger. Keep the remainder, if any, as part of the answer.

Proceed in the same manner with this and each succeeding quotient till the required denomination has been reached.

247. 4. Reduce 225932 inches to miles, etc.

5. How many miles and rods are there in 35640 ft. ?

6. Reduce 19922544 sq. in. to larger denominations.

7. Reduce 762051 cu. in. to cubic yards, etc.

8. Reduce 69056 oz. to tons, etc.

9. Reduce 21076 gr. to larger denominations.

10. Reduce 1947 gi. to gallons, etc.

11. How many bales in 24000 sheets of paper ?

12. Reduce 39180 min. to weeks, etc.

13. How many bushels, etc., in 35842 pints ?

14. How many pounds, etc. (Troy), in 32563 gr. ?

15. Reduce 39632 gr. to pounds, etc. (apoth.).

16. How many tons, etc., in 35682 lb. ?

17. A box contains 75832 pens. How many great gross, etc., in the box ?

18. Change 1384 dry pints to larger denominations.

19. In 139843 sq. in. how many square rods, etc. ?

20. Reduce 164808 in. to miles, etc.

21. In 12024 in. how many rods, etc. ?

248. Written Review.

1. How many 2-quart cups can be filled from a barrel of syrup?

2. Change 7 gal. 2 qt. 1 pt. to gills.

3. In 276 pints how many gallons?

4. A merchant paid \$10 for a barrel of molasses, and retailed it at \$.40 a gallon. What was his gain?

5. How many ounces are there in 250 lb.?

6. In $17\frac{1}{2}$ tons how many pounds?

7. How many bullets weighing $\frac{1}{2}$ oz. each can be made from 32 lb. 4 oz. of lead?

8. Reduce 118 lb. 7 oz. to drams.

9. Reduce 15 pk. 7 qt. to pints.

10. How many quarts are in 846 pints?

11. Henry gathered 16 bu. 2 qt. of walnuts, and sold them at \$.08 a quart. What did he receive for them?

12. In 8136 pt. how many bushels, etc.?

13. What will 6 bu. 2 pk. of cranberries be worth at \$.75 a peck?

14. A half-barrel of vinegar was sold at 2¢ a quart. What was received for it?

15. How many gallons in 16 quarts?

16. $13\frac{1}{2}$ gallons have been sold from a barrel. How many quarts are left?

17. At 7¢ a quart what will 15 gal. of vinegar cost?

18. How many quarts are there in 7 bu. 3 pk. 7 qt.?

19. A peck of peaches was sold from a crate containing a bushel. How many quarts were left?

20. How many bushels are there in 576 pints?

ADDITION OF COMPOUND NUMBERS

249. **1.** Add 14 lb. 5 oz. 17 pwt. 12 gr., 18 lb. 10 oz. 14 gr., 6 lb. 4 oz. 8 pwt. 16 gr.

lb.	oz.	pwt.	gr.
14	5	17	12
18	10	0	14
6	4	8	16
39	8	6	18

SOLUTION. — The sum of the grains = 42 gr. = 1 pwt. 18 gr. We place the 18 gr. under the column of grains, and add the 1 pwt. to the column of pennyweights. Add the other columns in like manner.

2.

rd.	yd.	ft.
17	4	1
12	4	2
6	5	$2\frac{1}{2}$
8	3	2
46	$1\frac{1}{2}$	$1\frac{1}{2}$
		$1\frac{1}{2} = \frac{1}{2}$ yd.
46	2	0

3.

rd.	ft.	in.
6	12	6
4	14	11
17	15	9
6	12	8
36	$5\frac{1}{2}$	10
		$6 = \frac{1}{2}$ ft.
36	6	4

4. Add:

bu.	pk.	qt.
7	1	3
10	1	2
4	1	2

5.

bbl.	gal.	qt.
6	7	1
5	12	1
4	6	1

6.

yd.	ft.	in.
1	2	6
2	1	
1	2	3

7.

bbl.	gal	qt.	pt.
4	7	4	1
6	2		1
3	5	1	1

Find the sum:

8. 3 bu. 2 pk. 2 qt. 1 pt.; 4 bu. 5 pk. 3 qt.; 7 bu. 1 pk. 4 qt. 1 pt.

9. 7 lb. 8 oz. 6 dr.; 4 lb. 11 oz. 5 dr.; 2 lb. 4 dr.

10. 1 bbl. 14 gal. 2 qt. 1 pt.; 2 bbl. 5 gal. 3 qt.; 7 gal. 3 qt.

SUBTRACTION OF COMPOUND NUMBERS

250. **1.**

	lb.	oz.	pwt.	gr.
From	6	2	14	15
Take	4	10	18	12
	1	3	16	3

SOLUTION. — 15 gr. − 12 gr. = 3 gr. As we cannot take 18 pwt. from 14 pwt., we take 1 oz., which equals 20 pwt., and add to the 14 pwt. = 34 pwt.; 34 pwt. − 18 pwt. = 16 pwt. We have taken 1 oz. from the 2 oz., leaving 1 oz.; but as we cannot take 10 oz. from 1 oz., we take 1 lb. = 12 oz., and add it to 1 oz. = 13 oz., from which take 10 oz. = 3 oz. Since we took 1 of the 6 lb., we have 5 lb. left; from which take 4 lb. = 1 lb.

2.

	A.	sq. rd.	sq. ft.
From	10	50	7
Take	4	106	5

3.

	da.	hr.	min.	sec.
	200	17	54	36
	135	20	24	48

4.

	hr.	min.	sec.
	5	54	30
	1	71	50

5.

	T.	cwt.	lb.	oz.
	20	15	75	10
	5	16	25	12

Subtract:

6. 16 lb. 8 oz. 3 dr.
5 4 2

7. 16 da. 5 hr. 36 min.
5 7 18

8. 7 yd. 3 ft. 9 in.
2 5 7

9. 27 gal. 3 qt. 1 pt. 2 gi.
18 2 1 3

10. From 6 dol. 7 di. 2 ct. 3 m. take 4 dol. 5 di. 8 ct. 1 m.

11. Find the difference between 4 lb. 8 oz. 3 dr. and 2 lb. 5 oz. 7 dr.

12. From a can containing 7 gal. 2 qt. 1 pt. of milk, 4 gal. 3 qt. were sold. How much was left?

13. 6 ft. 11 in. were cut from a pole 18 ft. 9 in. long. How long was the pole then?

251. 1. Find the time from Jan. 25, 1842, to July 4, 1896.

yr.	mo.	da.
1896	7	4
1842	1	25
54	5	9

It is customary to consider 30 days to a month. July 4, 1896, is the 1896th year, 7th month, 4th day of the Christian Era, and Jan. 25, 1842, is the 1842d year, 1st month, and 25th day. Subtract, taking 30 days for a month.

2. What is the exact number of days between Dec. 16, 1895, and March 12, 1896?

Dec.	15
Jan.	31
Feb.	29
March	12
	87 days *Ans.*

Do not count the first day mentioned. There are 15 days in December after the 16th. January has 31 days, February 29 (leap year), and March 12, making 87 days. Always count the last day.

3. Find the time between Dec. 11, 1620, and July 4, 1776.

4. How much time elapsed from the beginning of the Civil War, April 14, 1861, to the close of the war, April 9, 1865?

5. Washington was born Feb. 22, 1732, and died Dec. 14, 1799. How long did he live?

6. How much time since Oct. 12, 1492, to the present time?

7. Mr. Griffith gave a note dated Feb. 25, 1896, and paid it July 12, 1896. Find the exact number of days between the giving and the paying of the note.

8. Find the exact number of days between June 25, 1900, and Aug. 24, 1900.

Find the exact time between

9. Sept. 6, 1896, and April 7, 1897.

10. Nov. 11, 1898, and Dec. 4, 1898.

11. Aug. 16, 1900, and Dec. 21, 1900.

12. July 4, 1896, and Aug. 10, 1896.

13. Feb. 23, 1897, and June 4, 1897.

14. Oct. 9, 1899, and Feb. 6, 1900.

15. Nov. 8, 1894, and Oct. 6, 1895.

MULTIPLICATION OF COMPOUND NUMBERS

252. 1. Multiply 4 yd. 2 ft. 8 in. by 8.

yd.	ft.	in.
4	2	8
		8
39	0	4

SOLUTION.—8 times 8 in. = 64 in. = 5 ft. 4 in. Place the 4 in. under the inches column, and reserve the 5 ft. to be added to the product of 2 ft. by 8, which equals (adding 5 ft.) 21 ft. 21 ft. = 7 yd. with no remainder. Place 0 under the feet column and add 7 yd. to the product of 4 yd. by 8, which equals (adding the 7 yd.) 39 yd. The product, therefore, is 39 yd. 4 in.

2. 10 gal. 2 qt. 1 pt. 3 gi.
$\times$ 6

3. 12 bu. 2 pk. 5 qt. 1 pt.
$\times$ 8

4. How much hay in 6 loads if each load weighs 1 T. 2 cwt. 25 lb.?

5. What is the weight of 9 silver spoons if each weighs 3 oz. 14 pwt. 14 gr.?

6. How much oil in 5 barrels if each barrel contains 35 gal. 2 qt. 1 pt.?

7. What is the weight of 8 packages if each weighs 1 lb. 4 oz. (avoir.)?

8. A farmer has 6 bins, each containing 58 bu. 3 pk. 2 qt. How much wheat in the bins?

9. A farmer can plough one acre of ground in 7 hr. 20 min. 6 sec. At the same rate how long will it take him to plough 8 acres?

DIVISION OF COMPOUND NUMBERS

253. **1.** Divide 16 lb. 9 oz. 17 pwt. 7 gr. by 5.

	lb.	oz.	pwt.	gr.
5)	16	9	17	7
	3	4	7	11

SOLUTION. — $\frac{1}{5}$ of 16 lb. = 3 lb. and 1 lb. remaining. 1 lb. = 12 oz., which added to 9 oz. = 21 oz. $\frac{1}{5}$ of 21 oz. = 4 oz. with 1 oz. remaining. 1 oz. = 20 pwt., which added to 17 pwt. = 37 pwt. $\frac{1}{5}$ of 37 pwt. = 7 pwt. and 2 pwt. remaining. 2 pwt. = 48 gr., which added to 7 gr. = 55 gr. $\frac{1}{5}$ of 55 gr. = 11 gr.

The quotient, therefore, is 3 lb. 4 oz. 7 pwt. 11 gr.

2. Divide 54 bu. 3 pk. 3 qt. by 5.

3. 8 persons share equally in the contents of a bin containing 20 bu. 2 pk. of apples. What is the share of each?

4. When \$6 will buy 5 gal. 3 qt. 1 pt. of maple syrup, how much will \$1 buy?

5. If a horse eats 8 qt. of oats per day, how long will 10 bu. 1 pk. 5 qt. last him?

NOTE. — When both dividend and divisor are compound, reduce them to the same denomination and divide. The quotient will be abstract.

6. If a package weighs 4 cwt. 15 lb., how many such packages will it take to weigh 3 T. 2 cwt. 25 lb.?

7. Divide 102 T. 15 cwt. 27 lb. 8 oz. by 8.

8. I have 83 lb. 2 oz. of salt which I wish to put into packages of 2 lb. 6 oz. each. How many packages will there be?

9. 113 gal. 2 qt. 0 gi. $\div 4 = ?$

10. 126 gal. 3 qt. 1 pt. $\div 6 = ?$

11. 220 gal. 2 qt. $\div 7 = ?$

12. 6 T. 1200 lb. $\div 5 = ?$

13. Divide 746 oz. 4 pwt. by 7.

14. Divide 10 wk. 4 da. 3 hr. 8 min. by 4.

SURFACE MEASUREMENTS

254. A plane figure having four straight sides and four right angles is a rectangle.

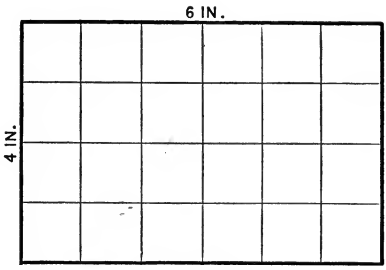
The **Area** of a surface is the number of square units that it contains.

What is the square unit in this rectangle?

How many square inches in it?

There are how many square inches in a row? There are how many rows?

Therefore how many square inches? How do you find it?



The length and breadth of a rectangle are called its Dimensions.

NOTE. — In finding areas, as in all multiplication, the multiplier is abstract, and the unit of the product must be the same as the unit of the multiplicand.

255. The answers to these questions give us the following:

Rule for Finding the Area of a Rectangle

Multiply the length by the breadth expressed in the same denomination. The result will be the area expressed in square units of the denomination corresponding to that of the dimensions.

Oral.

1. How many square feet in a surface 6 ft. long and 1 ft. wide?
2. In a surface 6 ft. long and 2 ft. wide?

3. How many square feet in a platform 5 ft. long and 4 ft. wide?

4. What is the area of a mirror 3 ft. long and 2 ft. wide?

5. Draw a rectangle 4 inches long by 2 inches wide, and divide it into square inches.

6. Draw a 2-inch square and divide it into square inches.

7. What is the difference between a 2-inch square and 2 square inches?

8. How many square feet does a 5-foot square contain? Show the difference between a 5-ft. square and 5 square feet.

9. If the top of a table is 5 ft. long and 4 ft. wide, what is its area?

10. If the area of a floor is 20 square yards, and the length 5 yards, what is the width?

Principles

1. Area is the product of length multiplied by breadth. The numbers representing length and breadth must be like numbers.

2. Area divided by either dimension gives the other.

256. Written.

1. How many square feet in the top of a table $5\frac{1}{2}$ ft. long and 4 feet wide? How many square inches?

2. What is the area of a basement 36 ft. by $18\frac{1}{2}$ ft.?

3. At 22¢ a square foot, what is the cost of a flag walk 50 ft. $\times$ 6 ft.?

4. How wide is a floor whose area is 300 sq. ft. and whose length is 20 ft.?

5. 5 flag-stones, each $5\frac{1}{2}$ ft. by 6 ft., will cover how much surface?

6. In a room are 4 doors, each 7 ft. 6 in. by 2 ft. 8. in. How many square feet in the surface of the four doors? How many square inches?

7. A pasture containing 4 acres is 20 rods wide. How long is it?

8. A city lot containing 8400 sq. ft. is 140 ft. deep. What is the frontage?

9. How many acres in a field $\frac{1}{2}$ mile square?

10. Is there any difference between a mile square and a square mile?

PLASTERING AND PAINTING

257. Plastering and painting are usually done by the square yard.

Deductions are frequently made for openings.

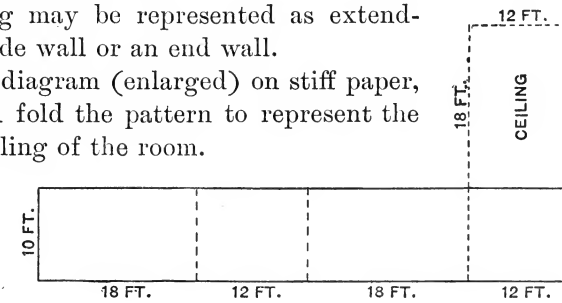
Windows and doors are openings.

1. A room is 18 ft. long, 12 ft. wide, and 10 ft. high. How many square yards in the walls and ceiling?

In problems of this kind first make a diagram representing the four walls in a line.

The ceiling may be represented as extending from a side wall or an end wall.

Draw this diagram (enlarged) on stiff paper, then cut and fold the pattern to represent the walls and ceiling of the room.



SOLUTION. — The length of the four walls is $2 \times (18 \text{ ft.} + 12 \text{ ft.}) = 60$ feet.

Area of four walls, 60 ft. by 10 ft. = 600 sq. ft.
Area of ceiling, 18 ft. by 12 ft. = 216 sq. ft.
Area of four walls and ceiling = 816 sq. ft. = $90\frac{2}{3}$ sq. yd.

2. If in the room mentioned above there are two doors, 3 ft. by 7 ft., and four windows 3 ft. by 6 ft., how many square yards remain in the four walls and ceiling?

Represent these doors and windows on the diagram.

3. How much will it cost at 25 cents a square yard to plaster the room mentioned in example 1, full deductions being made for openings?

4. What will it cost at $33\frac{1}{3}$ cents a square yard to plaster the walls and ceiling of a room 24 ft. long, 18 ft. wide, and 9 ft. high? The room has 3 doors 3 ft. by 7 ft., and 6 windows 3 ft. by 6 ft. One-half of the openings deducted.

5. How many square yards of plastering in a room 40 ft. long, 30 ft. wide, and 12 ft. high, deducting 80 sq. ft. for openings?

6. At 35¢ a sq. yard, what will it cost to plaster a room 18 ft. square and 9 ft. high, having 3 doors 7 ft. by 3 ft., and 5 windows 6 ft. by 3 ft., no allowance for openings?

CARPETING ROOMS

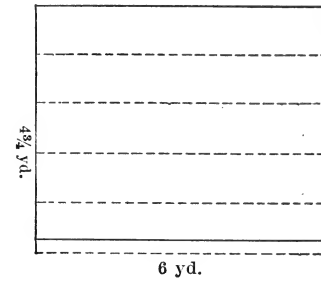
258. In making a carpet, the carpeting is cut from a roll into strips which are usually laid from end to end on the floor, or lengthwise. Sometimes the strips are laid across the room.

1. How much carpeting 1 yd. wide must I purchase to cover a room 6 yd. long and $4\frac{3}{4}$ yd. wide, strips running lengthwise?

SOLUTION. — It will be necessary to purchase as much carpeting as if the room were 5 yd. wide, the excess of $\frac{1}{4}$ yd. being turned under in the last strip.

1 strip contains 6 yd. 5 strips = 5 times 6 yd. = 30 yd. *Ans.*

2. How many yards must I purchase, if the strips are laid across the room?



SOLUTION. — 1 strip contains $4\frac{3}{4}$ yd. 6 strips = 6 times $4\frac{3}{4}$ yd. = $28\frac{1}{2}$ yd. *Ans.*

Carpeting is commonly 1 yd. or $\frac{3}{4}$ yd. wide.

NOTE. — It is often necessary to purchase more than enough carpeting to cover a room, on account of the waste in matching patterns.

This diagram represents the floor in example 1, in which the strips are laid lengthwise. Pupils should draw a similar diagram for each floor.

3. How many yards of carpet $\frac{3}{4}$ yd. wide will be required for a hall 18 ft. wide and 30 ft. long, the strips running the long way of the hall?

Make a diagram showing the strips or breadths.

How many linear yards in each strip?

How many strips?

SOLUTION. — Since one strip covers a space $\frac{3}{4}$ yd. wide, it will take as many strips to cover a space 6 yd. wide, as $\frac{3}{4}$ yd. is contained times in 6 yd. = 8 strips.

8 strips each 10 yd. long = 80 linear yards.

How many yards must be purchased if the strips run the short way?

4. At $1.25 a yard, what will it cost to carpet a hall $11\frac{1}{4}$ ft. wide and 28 ft. long, the carpet being 1 yard wide?

How many yards long is each strip?

How many strips?

5. How much would the carpet cost if there were a waste of $1\frac{1}{4}$ yd. for matching? (Ex. 4.)

6. How many yards of ingrain carpet 1 yd. wide will be required for a floor 17 ft. wide and 20 ft. long, strips running lengthwise?

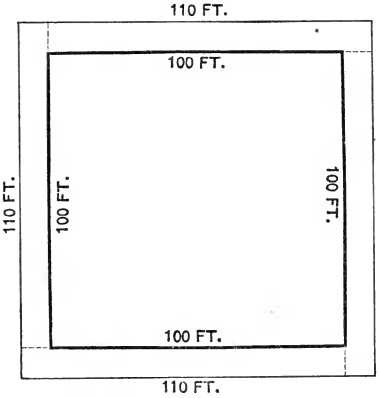
How many yards long is each strip?

How many strips would be needed if the room were 15 ft. wide? How many if it were 18 ft. wide?

It being 17 ft. wide, how many strips must be purchased?

How much of the last strip is to be turned under?

7. A sidewalk 10 ft. wide is to be laid on the 4 sides of a lot 100 ft. square. At 25¢ a sq. foot, what will be the cost?



The walk will be 110 ft. long on each of the four sides, as shown by the diagram. The entire length of the walk is 4 × 110 ft. = 440 ft.; this multiplied by the width of the walk will give its area.

8. What will be the cost of a 5-foot walk laid on the inside of the lot, close to the sides?

9. A man builds a house 45 by 35 ft. on a lot 60 by 150 ft. How many sq. yards has he left for a lawn?

10. What will it cost to build a tight fence 5 ft. high around the above lot at 25¢ a square yard?

11. What is the cost of a farm 80 rods square at $50 an acre?

12. How long is a rectangular field that contains 50 acres and is 80 rods wide?

PAPERING WALLS

259. A roll of paper is commonly 8 yards long and 18 inches wide.

A double roll is 16 yards long.

Borders are sold by the linear yard.

1. A room is 18 ft. by 15 ft., and 9 ft. high. Making no deduction for openings, how many square yards in the four walls and ceilings?

How many square yards will one roll cover?

How many rolls will be needed to paper the room?

If the room has 2 doors 3 ft. by $8\frac{1}{2}$ ft., and 4 windows 5 ft. by $6\frac{1}{2}$ ft., how many sq. yards of surface are to be covered?

How many rolls will cover it?

At $33\frac{1}{3}$ ¢ a roll, what will be the cost of papering?

At 50 ¢ a lineal yard, what will the border cost?

2. A hall is 12 ft. wide, 38 ft. long, and 10 feet high. The area of the openings is 117 sq. ft.

The paper costs 38 ¢ a roll.

The border costs $66\frac{2}{3}$ ¢ a yard.

Cost of putting on: 2 men $1\frac{1}{2}$ days each at $2 a day.

Full deductions made for openings.

Find the entire cost and make the bill.

3. A sitting room is 20 ft. by 17 ft., and 10 ft. high. It has 3 doors 2 ft. 6 in. by 7 ft., and 4 windows $3\frac{1}{4}$ ft. by 6 ft. How many rolls of paper will cover the walls and ceiling, deducting $\frac{1}{2}$ the area of the openings?

BOARD MEASURE

260. A **Board Foot** is a square foot of the surface of a board 1 inch thick, or less.

A board 10 ft. long, 1 ft. wide, and 1 in. thick, or less, contains 10 board feet; but a beam 10 ft. long, 1 ft. wide, and 8 in. thick contains 8 times 10 board feet, or 80 board feet.

The number of board feet = Length (in feet) × Width (in feet) × Thickness (in inches).

NOTE. — When the thickness is 1 in., or less, the number of board feet is the product of the length and width in feet.

1. How many board feet in a board 16 ft. long, 12 in. wide, and 1 in. thick?

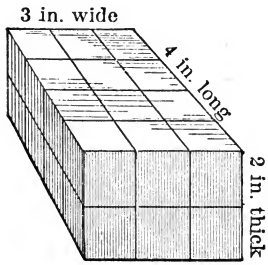
2. What will be the cost of 10 planks, each 12 ft. long, 10 in. wide, and 3 in. thick, at $12 a thousand?

3. How many board feet in a board 14 ft. long, 14 in. wide, and $\frac{3}{4}$ in. thick?

4. How many board feet in 6 joists 16 ft. long, 12 in. wide, and 3 in. thick?

VOLUME MEASUREMENTS

261. The **Volume** or **Solidity** of a body is the number of cubic units that it contains.



How many cubic inches in a block 4 inches long, 3 inches wide, and 2 inches thick?

How many rows of cubic inches are there in the bottom layer? How many cubic inches in each row? How many cubic inches in the layer? How do you find it? How many layers? How many cubic inches in the 2 layers? How do you find it?

262. The answers to these questions give us the following

Rule for Finding the Volume of a Solid

Multiply together the length, breadth, and thickness, all expressed in the same denomination.

The result will be the volume expressed in cubic units corresponding to the unit of the dimensions.

1. How many cubic inches in a block 12 inches long, 6 inches wide, and 4 inches thick?

2. What is the volume of a block of marble 6 ft. long, 5 ft. wide, and 4 ft. thick?

3. What is the volume of 6 bricks, each being 8 in. by 4 in. by 2 in.?

4. The solid contents of a block are 720 cubic feet. It is 10 ft. high and 9 ft. wide. How thick is it?

NOTE. — To find one dimension when the solidity and the other two dimensions are known, divide the solidity by the product of the two given dimensions.

5. How many cubic inches of space in a box 10 in. long, 6 in. wide, and 4 in. high?

What part of a cubic yard is the space in this box?

6. A piece of marble 16 ft. long, 12 ft. wide, and 9 ft. thick can be sawed into how many blocks of a cubic foot each?

7. What will it cost to make an embankment containing 54,270 cu. ft. of earth at $.15 a cubic yard?

8. Find the number of cubic inches in a piece of ice 3 ft. long, $2\frac{1}{2}$ ft. wide, and 18 inches thick.

9. What is the difference between 6 cubic inches and a 6-inch cube?

10. If 300 cu. ft. of breathing space is required for a pupil, how many pupils should be seated in a schoolroom 30 ft. square and 10 ft. high?

11. How many cubic yards of earth will be removed in digging a cellar 36 ft. square and 6 ft. deep?

12. How many books 9 in. by 6 in. by $1\frac{1}{2}$ in. can be packed into a chest 4 ft. by 2 ft. by $1\frac{1}{2}$ ft.? (Indicate and cancel.)

13. How many cubic feet of snow on 18 square feet of ground, the snow being 9 inches deep?

14. How many bricks, $8 \times 4 \times 2$ inches, will it take to build a wall $16 \times 8 \times 1\frac{1}{4}$ ft., no allowance for mortar? (Cancel.)

15. How many cubic inches in a stick of timber 1 foot square at the ends and 8 ft. long?

16. How many 2-inch cubes may be made from a piece of wood 6 ft. by 4 ft. by 2 ft.?

17. How many loads of earth, each load containing a cubic yard, in an excavation 30 ft. long, 26 ft. wide, and 14 feet deep?

18. A cubic yard of earth makes a load. How many loads must be excavated for a cellar 32 ft. long, 30 ft. wide, and $6\frac{1}{2}$ ft. deep?

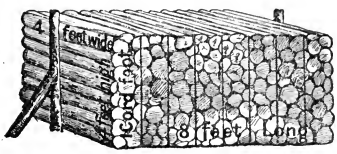
19. A brick is 8 in. long, 4 in. wide, and 2 in. thick. How many will it take to build a wall 30 ft. long, 20 ft. wide, and $1\frac{1}{2}$ ft. thick, making no allowance for mortar?

20. The volume of a rectangular solid is 1200 cu. ft. It is 20 ft. long and 4 ft. high. How wide is it?

21. How high is a room that is $24\frac{1}{2}$ ft. long, 20 ft. wide, and contains 4900 cubic feet?

WOOD MEASURE

263. A pile of wood 4 feet wide, 4 feet high, and 1 foot long is a **Cord Foot.** It contains how many cubic feet?



A pile of wood 4 feet wide, 4 feet high, and 8 feet long contains how many cord feet? cubi feet

264. The answers to these questions give us the following

TABLE OF WOOD MEASURE

16 cubic feet	make 1 cord foot (cd. ft.).
8 cord feet or 128 cubic feet	make 1 cord (cd.).

How many cords of wood in the following?

1. A pile 18 ft. long, 4 ft. wide, 4 ft. high.

2. A pile 20 ft. long, 8 ft. wide, 8 ft. high.

3. A pile 50 ft. long, 12 ft. wide, and 17 ft. high.

4. What is the cost of a pile of wood 10 ft. long, 8 ft. wide, and 8 ft. high at $4.50 a cord?

5. How high must a pile of wood be made to contain 20 cords, if the pile is 20 ft. long?

6. How many cords of building stone in a pile 18 ft. long, 6 ft. wide, and 4 ft. high?

7. How many cords of wood can be piled into a shed 24 ft. long, 18 ft. wide, and 12 ft. high?

CAPACITY

265. Capacity of Cisterns.

1. A wine gallon fills 231 cubic inches of space. How many gallons will fill a cubic foot of space?

$\frac{1728}{231}$ = about $7\frac{1}{2}$ gallons. *Ans.*

NOTE. — It is sufficiently accurate in estimating large liquid capacities to reckon $7\frac{1}{2}$ gal. to the cubic foot.

2. A cistern is 5 ft. square and 6 ft. deep. How many gallons of water will it contain?

Solve both ways and note the difference.

3. What is the capacity in barrels of the cistern in Ex. 2?

4. How many barrels of water in a cistern 7 ft. long, $4\frac{1}{2}$ ft. wide, and $5\frac{1}{2}$ ft. deep? (Indicate and cancel.)

5. What is the capacity in barrels of a tank 3 ft. by $4\frac{1}{2}$ ft. and $4\frac{3}{4}$ ft. deep?

6. How many gallons of water in a tank whose capacity is 150 cubic feet?

7. How many cubic feet in a cistern that will hold 1500 gallons of water?

266. Capacity of Bins.

8. A bushel fills 2150.42 cubic inches of space, which is $1\frac{1}{4}$ cu. ft., nearly.

NOTE. — In estimating large quantities of grain it is sufficiently accurate to reckon $1\frac{1}{4}$ cu. ft. to the bushel.

9. How many bushels of grain will a bin hold that is 6 ft. long, 5 ft. wide, and 4 ft. deep? (Indicate and cancel.)

Solve both ways and note the difference.

10. How many bushels in a bin that contains 150 cubic feet?

11. How many cubic feet in a bin that holds 150 bushels?

PERCENTAGE

267. To the Teacher. — Before teaching percentage review thoroughly Questions of Relation and Division of Decimals.

Questions of Relation may be solved by means of hundredths.

1. How much is $\frac{25}{100}$ of 20? *Ans.* 5.
2. 5 is $\frac{25}{100}$ of what? *Ans.* 20.
3. 5 is how many hundredths of 20? *Ans.* $\frac{25}{100}$.

Another name for *hundredths* is *per cent.* Thus, $\frac{25}{100}$ is 25 per cent, $\frac{8}{100}$ is 8 per cent, .16 is 16 per cent, .05 is 5 per cent.

Per cent means by the hundred or on the hundred.

The sign of per cent is %. 25 per cent is 25%, 6 per cent is 6%, 50 per cent is 50%.

In performing operations *per cent* is used generally as decimal hundredths.

Using the sign %, Examples 1, 2, and 3 become as follows:

Note. — These questions may be solved as in Art. 177.

a. How much is 25% of 20?

$20 \times .25 = 5$. Therefore 25% of $20 = 5$.

b. 5 is 25% of what number?

Since $20 \times .25 = 5$ $5 \div .25 = 20$. Therefore 5 is 25% of 20.

c. 5 is what per cent of 20?

Since $20 \times .25 = 5$, $5 \div 20 = .25$. Therefore 5 is 25% of 20.

What principle is used in solving questions *b* and *c*?

Question *a*

Find the result and form questions *b* and *c*:

4. How much is 8% of 50?
5. How much is 5% of 200?
6. How much is 12% of 150?
7. 10% of 60 sheep are how many sheep?
8. 50% of 300 men are how many men?
9. 2% of 30 bushels are how many bushels?

Question *b*

Find the result and form questions *a* and *c*

10. 16 is 25% of what number?
11. 15 is 10% of what number?
12. 210 is 1% of what number?
13. 20 sheep are 5% of how many sheep?
14. 40 pupils are 10% of how many pupils?
15. 80 horses are 16% of how many horses?

Question *c*

Find the result and form questions *a* and *b*.

16. 20 is what % of 80?
17. 10 is what % of 200?
18. 50 is what % of 500?
19. 25 boys are what per cent of 100 boys?
20. 15 pounds are what per cent of 60 lb.?
21. What per cent of 80 men are 40 men?

Find the result, form the other two questions, and solve each.

22. 5% of 40 apples are how many?

23. \$15 is 10% of what?

24. 12 yd. is what % of 48 yd.?

25. How much is 60% of 200 bushels?

26. 40 men are 5% of how many men?

268. Percentage is a process of solving questions of relation by means of hundredths.

269. Read the following Questions of Relation:

Question *a*. How much is 5% of 200? *Ans.* 10.

Question *b*. 10 is 5% of what? *Ans.* 200.

Question *c*. 10 is what % of 200? *Ans.* 5%.

These three kinds of questions form the basis of a great variety of practical computations, which are classed under the general head of Percentage.

270. Every question in percentage involves three elements: the **Rate** per cent, the **Base**, and the **Percentage**.

271. The **Rate per Cent** is the number of hundredths taken. In question *a*, what is the rate per cent?

272. The **Base** is the number of which the hundredths are taken. In question *a*, what is the base?

273. The **Percentage** is the result obtained by taking a certain per cent of a number. In question *a*, what is the percentage?

How much is 8% of \$200?

SOLUTION. — 8% of \$200 = 200 × .08 = \$16. We now have the three elements, as follows:

8% is the rate, \$200 is the base, and \$16 is the percentage.

Since \$200 × .08 = \$16, the percentage;

\$16 ÷ .08 = \$200, the base;

and \$16 ÷ \$200 = .08, the rate.

274. Therefore, when any two of these elements are given, the other may be found, thus :

Base × Rate = Percentage ;
Percentage ÷ Rate = Base ;
Percentage ÷ Base = Rate.

275. Tell which elements are given, and which one is required, in question *a* ; in question *b* ; in question *c*.

To find the Percentage when the Base and the Rate are given, multiply the Base by the Rate.

1. A boy had $4.00, and spent 10% of it for a book. How much did the book cost ?

To the Teacher. — In solving any problem in percentage, the pupil should first state the question. The question in Ex. 1 is "How much is 10% of $4.00 ?"

2. A farmer had 100 sheep and sold 20% of them. How many did he sell ? (Write the question, then solve.)

3. A man having 500 acres of land gave 20% to his son. How many acres did his son receive ? (Question.)

4. If I buy goods for $400 and sell them at a loss of 5%, how much do I lose ? (Question.)

5. Write a problem having $500 for the base and 4% for the rate.

To find the base when the Percentage and the Rate are given, divide the Percentage by the Rate.

6. \$40 is 10% of what?

Solve, then state questions *a* and *c*.

7. A man sold goods at 50% profit, and thereby gained \$200. What was the cost?

First state the question. (\$200 is 50% of what?)

8. A boy paid 40 cents for a book, which was 10% of his money. How much money had he? (Question.)

9. A man sold 20 sheep, which was 20% of his flock. How many sheep were there in the flock? (Question.)

10. 10% of my salary is \$160. What is my salary? (Question.)

11. By selling cotton at 5% below cost I lose \$200. What was the cost of the cotton? (Question.)

12. A grocer gained \$50 by selling goods at 10% profit. What did the goods cost? (Question.)

13. I sold a bicycle at an advance on the cost of 20% and thereby gained \$10. What did I pay for the bicycle? (Question.)

14. A man pays \$300 rent, which is 20% of his salary. What is his salary? (Question.)

> To find the Rate when the Percentage and the Base are given, divide the Percentage by the Base.

15. \$50 is what per cent of \$500?

Solve, then state questions *a* and *b*.

16. By selling a bicycle that cost \$50, a dealer made a profit of \$10. What was his gain per cent? First state the question. (\$10 is what per cent of \$50?)

17. In a school of 900 pupils 450 are girls. What per cent of the pupils are girls? (Question.)

18. A boy had 80 cents and spent 20 cents. What per cent of his money did he spend? (Question.) What per cent of his money did he save? (Question.)

19. From a farm containing 400 acres 32 acres were sold. What per cent of the farm was sold? (Question.)

20. A farmer had 200 sheep and sold 50 sheep. What per cent of his sheep were sold? (Question.) What per cent were not sold? (Question.)

21. In a spelling lesson of 20 words Lucy misspelled 5. What per cent did she misspell? (Question.) What per cent did she spell correctly? (Question.)

SIMPLE INTEREST

276. 1. I borrow $500 for 1 year, and at the end of the year I repay the money and 6% for the use of it. How much do I pay for the use of $500?

2. How much must be paid for the use of $50 for 1 year at 5%? at 7%?

3. How much at 5% per annum must I pay for the use of $1000 for 1 year? for 3 years?

4. I loan James Barnes $500 at 6%. At the end of 2 years he pays me in full. How much does he pay me?

277. Money that is paid for the use of money is called **Interest.** The money for the use of which interest is paid is called the **Principal,** and the sum of the principal and interest is called the **Amount.**

Interest at 6% means 6% of the principal for 1 year.

12 months of 30 days each are usually regarded as a year in computing interest.

Oral. **5.** What is the interest of $100 for 3 years at 6%?

SOLUTION.—

$100	Principal.
.06	Rate.
$6.00	Interest for 1 year.
3	
$18.00	Interest for 3 years.

6. What is the interest of $80 at 5% for $2\frac{1}{2}$ years?

7. What is the interest of $1000 at 5% for 2 yr. 6 mo.?

8. What is the interest of $100 at 6% for 1 year? For $1\frac{1}{2}$ year? For 2 yr. 6 mo.? For 3 yr. 3 mo.?

278. When the time does not include days, find interest as follows:

Rule for Finding Interest

Multiply the principal by the rate per annum and that product by the time in years.

9. What is the interest of $297.62 for 5 yr. 3 mo. at 6%?

SOLUTION.—

$297.62	
.06	
$17.8572	
$5\frac{1}{4}$	
44643	
892860	
$93.75.	*Ans.*

NOTE.—Final results should not include mills. Mills are disregarded if less than 5, and called another cent if 5 or more.

Find the interest of:

10. $384.62 at 6% for 2 yr.

11. $463.75 at 7% for 3 yr.

12. $250.50 at 8% for 5 yr.

13. $685.20 at 4% for 6 yr.

14. $596.15 at 5% for 2 yr. 3 mo.

15. $386.42 at 5% for 6 yr. 5 mo.
16. $950.16 at 10% for $4\frac{1}{2}$ yr.
17. $283.25 at 6% for 2 yr. 8 mo.

Find the amount of:

18. $284.10 for 3 yr. 2 mo. at 7%.
19. $364.24 for 1 yr. 1 mo. at 6%.
20. $282.50 for 5 yr. 9 mo. at 5%.
21. $298 for 4 yr. 3 mo. at 6%.
22. $389 for 7 yr. 10 mo. at 5%.

279. When the time includes days, we find interest by the following

Rule for Finding Exact Interest

To compute exact interest, find the exact time in days and consider 1 day's interest as $\frac{1}{365}$ of 1 year's interest.

1. Find the exact interest of $358 for 74 days at 7%.

SOLUTION. — $358 × .07 = $25.06, 1 year's interest. 74 days' interest is $\frac{74}{365}$ of 1 year's interest. $\frac{74}{365}$ of $25.06 = $5.08. *Ans.*

Find the exact interest of:

2. $324 for 15 d. at 9%.
3. $253 for 98 d. at 4%.
4. $624 for 117 d. at 7%.
5. $153.26 for 256 d. at 5%.
6. $620 from Aug. 15 to Nov. 12 at 6%.
7. $540.25 from June 12 to Sept. 14 at 8%.
8. $7560 for 90 days at 5%.
9. Find the exact interest at 5% on a note dated Jan. 14, 1896, and paid March 31, 1896, for $832.

TOPICAL REVIEW

280. 1. Write in words 5007.

2. Write in figures fourteen thousand seventeen.

3. Write in figures CCXI.

4. Write in Roman 245.

5. Add:

$$\begin{array}{r} 326 \\ 48 \\ 17 \\ 9 \\ 203 \\ 74 \\ 28 \\ 39 \\ 837 \\ 950 \\ 706 \\ \hline \end{array}$$

6. From twenty-four thousand eight hundred eight take twelve thousand nine hundred nine.

7. If you have 24 cents and spend 8 cents for a tablet and 5 cents for a pencil, how many cents have you left? Indicate the operation by signs.

8. Divide 35,702 by 53.

9. Write two prime numbers.

10. What is the quotient of 24×56 divided by 7×12? Indicate and cancel.

COMMON FRACTIONS

281. 1. Write an improper fraction whose value is more than 1. Write an improper fraction whose value is 1.

2. Change $\frac{4}{5}$ to a fraction whose denominator is 40.

3. Change to lowest terms $\frac{56}{944}$.

4. How many eighths in fifteen?

5. Write two like fractions.

6. Change to common denominator and add: 9, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{12}$.

7. $\$14\frac{1}{2} + \$9\frac{3}{4} - \$16\frac{1}{2} =$ what? Write a problem using these numbers.

8. The sum of two fractions is $\frac{16}{20}$. One of them is $\frac{2}{5}$. What is the other?

9. $15\frac{2}{5} - 5\frac{1}{8} = ?$ $3\frac{8}{9} \times 5\frac{1}{2} = ?$ $28\frac{1}{2} \div 2\frac{7}{8} = ?$

10. Add: $28\frac{3}{4}$, $17\frac{1}{2}$, $15\frac{2}{3}$, $8\frac{1}{4}$.

282. 1. Write a proper fraction, an improper fraction, a mixed number, and an integer, and use them in one problem.

2. Write a complex fraction.

3. What is a factor of a number? What is the greatest common divisor of two or more numbers?

4. If $\frac{1}{5}$ of my money is invested in land, $\frac{3}{4}$ in houses, and the remainder is deposited in the bank, what fraction of my money is in the bank?

5. Find the value of $\dfrac{3\frac{3}{4}}{\frac{2}{3} \text{ of } \frac{5}{6}} + \frac{1}{2}$ of $\frac{1}{3} \div \frac{2}{3}$.

6. Reduce to lowest terms $\frac{8}{9}\frac{5}{3}\frac{6}{6}$.

7. The sum of two fractions is $\frac{13}{18}$. One of them is $\frac{3}{7}$. What is the other?

8. A man owned $\frac{2}{3}$ of a ship and sold $\frac{1}{4}$ of his share. What part of the ship did he sell?

9. A man had \$500 in the bank. He drew out $\frac{3}{5}$ of it. How much did he draw? (Question.)

10. A man sold 50 sheep, which was $\frac{5}{7}$ of his entire flock. How many sheep in the flock? (Question.)

DECIMAL FRACTIONS

283. 1. In the number 32.6, what would be the value of the 2 if it were removed one place to the right? What would be its value if removed one place to the left?

2. Which is the greater, 50 thousandths or 5 hundredths?

3. Take one thousandth from one thousand.

4. Multiply $2\frac{1}{2}$ by 2.84.

5. Multiply 100 by .01.

6. $1.111 \div .01 = ?$ $1 \div .1 = ?$ $.1 \div 1 = ?$

7. Change to decimals: $\frac{3}{5}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{5}{8}$.

8. Write .0000054 as a common fraction.

9. Write as a decimal fraction, thirty-four, and three thousand four hundred six ten-millionths.

10. Write as a decimal, 15 5-10000.

284. 1. Make out a receipted bill of the following: 66 lb. pork at $12\frac{1}{2}¢$; 148 lb. veal at $16\frac{2}{3}¢$; 48 boxes strawberries at $16\frac{2}{3}¢$; 60 doz. eggs at $16\frac{2}{3}¢$.

Mr. A is the buyer, and Mr. B the seller. In multiplying use the aliquot parts of \$1.

2. How do you locate the decimal point in any product? In any sum?

3. Change $62\frac{1}{2}$ and $83\frac{1}{3}$ each to a common fraction.

4. Divide 28.78884 by 1.25 and have four decimal figures in the quotient.

5. Divide .001 by 1000.

6. What cost 1650 oysters at 50 cents a hundred?

7. What is the cost of 3783 ft. of pine lumber at \$16 a thousand?

8. What is the cost of 7384 tons of coal at \$5 a ton?

285. 1. What is the cost of 80 tablets at $16\frac{2}{3}¢$ each? (Use the aliquot part.)

2. At $33\frac{1}{3}¢$ a yard, how many yards of linen can be bought for \$9? (Use the aliquot part.)

3. In the number 66.6, what is the value of the first 6? The second 6? The third 6?

4. Which is the greater, $\frac{2}{3}$ or $\frac{3}{4}$, and how much?

5. John spent .75 of his money. What part of his money had he left?

6. Find the cost of these articles purchased by J. D. Fox from John Crouse Jan. 10, 1882, and make out the bill in proper form:

4 lb. coffee at 25¢ a pound.
18 lb. sugar at $6\frac{1}{4}$¢ a pound.
4 gal. of molasses at 50¢ a gallon.
16 lb. of rice at $8\frac{1}{4}$¢ a pound.

7. How many bushels of wheat at $.90 a bushel must be given for 9 pieces of cloth, each containing 40 yards at $.50 a yard? (Cancellation.)

8. The lesser of two numbers is 4603.85 and their difference is 835.007. What is the greater number?

9. John gave away 30 of his marbles. How many remained if he had 200 marbles at first?

10. 200 is $\frac{25}{100}$ of what?

DENOMINATE NUMBERS

286. **1.** How many bushels of apples at 50¢ a bushel must be given for 15 lb. of tea at 40¢ a lb.? (Cancellation.)

2.

From	7 mi.	40 rd.	9 yd.
take	4 mi.	17 rd.	8 yd.

3. How long is a fence that extends around a field 124 rd. long and $78\frac{1}{2}$ rd. wide? (Draw diagram of field and label each side.)

4. Define Compound Number and write one.

5. If a peck measure is one-quarter full, how many quarts does it contain?

6. How many bushels in 1152 pints?

7. Three quarts of milk are taken from a two-gallon pail. How many quarts remain?

8. How many tons in 6000 pounds?

9. What will $4\frac{1}{2}$ lb. of indigo cost at \$.05 an ounce?

10. What will it cost to build 15 yd. of fence at 58¢ a foot?

287. 1. How much rope is there in 3 balls, if there are 20 yd. 1 ft. in one, 18 yd. 1 ft. 8 in. in another, and 9 yd. 2 ft. 11 in. in the third?

2. At \$2.40 a dozen, what will 8 handkerchiefs cost?

3. John had \$25 and spent \$15 for a suit of clothes. What part of his money did he spend? (Question.)

4. The floor of a room 12 ft. wide is covered by 24 yd. of carpet. How long is the room?

5. At $\$\frac{5}{8}$ a yard, how many yards of cloth can be bought for \$25? (Solve by cancellation.)

6. Reduce 25 sq. yd. 6 sq. ft. to square inches.

7. From $1\frac{1}{2}$ sq. miles of land I sold a farm 160 rd. long by 100 rd. wide. How many acres are left?

8. Reduce 36,940 pints to higher denominations.

9. How many feet in a mile?

288. 1. How many acres in 640 sq. rods?

2. Show the difference between a 3-foot square and 3 square feet.

3. A rectangle contains 320 sq. yards and is 20 yards long. How wide is it? (Make a drawing of the rectangle and label the sides.)

4. At $18 a ton, what is the cost of 3000 lb. of hay?

5. At $\$\frac{3}{8}$ a pound, what will 2 pounds of tea cost?

6. What cost 3 lb. 8 oz. of butter at 30¢ a pound?

7. I bought 8 cwt. 10 lb. of sugar for $81 and sold it at 12¢ a pound. What was the gain?

8. Find the area of a floor $16\frac{1}{2}$ ft. long and 4 yd. wide.

9. How many gallons of water will a cistern contain that is 7 ft. long, $4\frac{1}{2}$ ft. wide, and $4\frac{3}{4}$ ft. deep?

10. How many bushels of grain will a bin hold that is 6 ft. × 5 ft. × 4 ft.?

289. 1. How high is a room that is $24\frac{1}{2}$ ft. long, 20 ft. wide, and contains 4410 cubic ft.?

2. A square mile of land may be cut into how many 80-acre farms? (Show by a diagram as well as by figures.)

3. Change $\frac{1}{4}$ of a ton to pounds.

4. 3 oz. is what part of a pound?

5. Reduce to higher denominations 369,426 cu. inches.

6. How many cords of wood can be piled into a shed 16 ft. by 10 ft. by 8 ft.? (Indicate and cancel.)

7. How many cords in a pile of wood 86 ft. 3 in. long, 7 ft. 6 in. high, and 4 ft. wide?

8. How many bushels will a bin contain that is 9 ft. long, 4 ft. wide, and 6 ft. deep?

9. Find the cost of $9\frac{1}{2}$ tons of coal, if $\frac{3}{4}$ of a ton cost $3.00.

10. How many rods of fence are required to enclose a lot 20 rods wide and 3 times as long?

ANSWERS TO HEATH'S PRIMARY ARITHMETIC

Article 31. — **4.** 438. **5.** 395. **6.** 495. **7.** 413. **8.** 575. **9.** 455. **10.** 347. **11.** 3196. **12.** 3846. **13.** 3549. **14.** $2480.45. **15.** 469. **16.** 4139. **17.** 5354. **18.** $549.88. **19.** $732.98. **20.** 6115. **21.** 8915. **22.** 18,441. **23.** 21,365. **24.** $771.40. **25.** $1287.873. **26.** $821.191. **27.** $577.189. **28.** 890,407. **29.** 103,019. **30.** $10,290.28. **31.** 1364. **32.** $240.75. **34.** 31,014. **35.** 273,426. **36.** 875,935. **37.** 51,543. **38.** $41,981.17. **39.** $136.09. **40.** 373,915. **41.** 176,880. **42.** 651,021. **43.** $511,233.

Article 32. — **1.** 41 peaches. **2.** 41 scholars. **3.** 98 sheep. **4.** 87 bu. **5.** $1.30. **6.** 101 trees. **7.** 80 qt. **8.** 133 mi. **9.** 126 fowls. **10.** $2.20. **11.** 62 years. **12.** 71 houses. **13.** 52 minutes. **14.** 43 fish. **15.** 2057 miles. **16.** $54.05. **17.** 521 bu. **18.** 658 pages.

Article 43. — **1.** 2145. **2.** 6927. **3.** 3698. **4.** 5499. **5.** 1474. **6.** 27,997. **7.** 18,338. **8.** 17,082. **9.** 81,482. **10.** 68,004. **11.** 4134. **12.** 71,359. **13.** 8422. **14.** 11,036. **15.** 5347. **16.** 8983. **17.** $1.032. **18.** $8.233. **19.** $24.025. **20.** $19.687. **21.** $10. **22.** $12.082. **23.** $1.052. **24.** $50.68. **25.** 7529. **26.** 25,350. **27.** 21,996. **28.** 3252. **29.** 6039. **30.** 4127. **31.** $2341.64. **32.** $80.308. **33.** 7726. **34.** 11,369. **35.** $23.967. **36.** 41,276. **37.** $813.875. **38.** $2.33. **39.** 166 acres. **40.** $149.36. **41.** 27 gal. **42.** 73 oranges. **43.** 163 acres. **44.** 36 yd. **45.** 49 sparrows. **46.** $125. **47.** 63 cents. **48.** 95 eggs. **49.** 27 qt.

Article 44. — **1.** 237 bu. **2.** 39 mi. **3.** $174. **4.** $1.99. **5.** $4.35. **6.** 252 mi. **7.** $268. **8.** $.43. **9.** 83 papers. **10.** $99. **11.** 447 pupils. **12.** $151. **13.** 15 years. **14.** $8.01. **15.** 61. **16.** 132 sheep. **17.** 410 bu. **18.** 162. **19.** 59 acres.

Article 61. — **1.** 7776. **2.** 11,544. **3.** 17,415. **4.** 14,728. **5.** 56,196. **6.** 54,600. **7.** 33,432. **8.** 96,728. **9.** 15,996. **10.** 34,314. **11.** 79,784. **12.** 21,528. **13.** 14,553. **14.** 79,001. **15.** 8288. **16.** 27,066. **17.** 11,124. **18.** $788.20. **19.** $944.46. **20.** $912.87. **21.** $973.648. **22.** $431.31. **23.** $1661.175. **24.** $11,042.21. **25.** $857.088. **26.** $962.066. **27.** $7285.185. **28.** $1046.464. **29.** $10,098.72. **30.** $9967.12. **31.** $73,671.50. **32.** $16,252.81. **33.** $63,677.16.

Article 62. — **1.** 62,000. **2.** 516 sheep. **3.** 392 pounds. **4.** $504. **5.** $3.60. **6.** 2880 rods. **7.** 8800 yd. **8.** $17.50. **9.** 583 mi. **10.** 384 trees. **11.** $15. **12.** $19.35. **13.** 784 pounds. **14.** $8.10. **15.** $63. **16.** $105. **17.** 36,960 ft. **18.** 20,000 pounds. **19.** $2304. **20.** $10.32. **21.** $1926. **22.** 5630. **23.** 870 acres. **24.** $49.50. **25.** 1728 pens. **26.** 2616.

Article 73. — **1.** 28. **2.** $62\frac{8}{18}$. **3.** $113\frac{18}{21}$. **4.** $28\frac{9}{17}$. **5.** $52\frac{13}{15}$. **6.** $50\frac{1}{27}$. **7.** $31\frac{11}{31}$. **8.** $12\frac{19}{22}$. **9.** $59\frac{1}{21}$. **10.** $99\frac{9}{13}$. **11.** $71\frac{15}{19}$. **12.** $123\frac{8}{32}$. **13.** $80\frac{9}{16}$. **14.** $153\frac{22}{37}$. **15.** 302. **16.** $32\frac{38}{39}$. **17.** $71\frac{1}{12}$. **18.** $40\frac{8}{25}$. **19.** $200\frac{36}{47}$. **20.** $223\frac{24}{46}$. **21.** $34\frac{19}{46}$. **22.** $80\frac{29}{51}$. **23.** $47\frac{16}{57}$. **24.** $79\frac{3}{19}$. **25.** $360\frac{3}{27}$. **26.** $100\frac{14}{39}$. **27.** $90\frac{59}{62}$. **28.** $17\frac{18}{73}$. **29.** $195\frac{48}{84}$. **30.** $215\frac{12}{52}$. **31.** $18\frac{34}{72}$. **32.** $74\frac{17}{86}$. **33.** $539\frac{10}{24}$. **34.** $101\frac{62}{91}$. **35.** $99\frac{32}{78}$. **36.** $241\frac{1}{45}$. **37.** $595\frac{11}{36}$. **38.** $283\frac{27}{37}$. **39.** $673\frac{33}{55}$. **40.** $667\frac{19}{60}$.

Article 83. — **1.** 360 mi. **2.** 178 bu. **3.** 45 cents. **4.** 153 tons. **5.** 54 rd. **6.** 82 da. **7.** 109. **8.** 84 ploughs. **9.** 202 acres. **10.** $4.625. **11.** 415 pk. **12.** 52 wk. **13.** $3287. **14.** $4.20. **15.** 1760 yd. **16.** 125 overcoats. **17.** 38 hours. **18.** 72. **19.** 36 gal. **20.** 120 tons. **21.** 106 acres. **22.** 24 stoves. **23.** $33\frac{1}{3}$ da. **24.** 49 wk.

Article 85. — **1.** 56. **2.** 14. **3.** 70 cents. **4.** 16 hr. **5.** 24 hr. **6.** $20.83. **7.** $76.80. **8.** $22.14. **9.** $42. **10.** $14\frac{38}{59}$. **11.** 90 da. **12.** $244. **13.** 656 mi. **14.** $3924. **15.** 340 miles. **16.** $4\frac{24}{43}$ pounds. **17.** $766. **18.** $965. **19.** 38 times. **20.** 55 months. **21.** 12. **22.** 5 years. **23.** $159.33. **24.** 33 tons. **25.** 16 tablets. **26.** Carriage 182, horse 183. **27.** $72. **28.** Edward $4.68, Henry $2.93, John $5.19. **29.** $13. **30.** $65.28. **31.** $50. **32.** 960 sheets. **33.** 259 sheep. **34.** 40 farms. **35.** $192.40. **36.** $1590. **37.** 24 acres. **38.** $3.69. **39.** $11.50. **40.** $48\frac{6}{13}$ pounds. **41.** 46 miles an hour. **42.** 264. **43.** $469. **44.** $284.25. **45.** $41. **46.** 70 hours.

Article 97. — **1.** $2 \cdot 5 \cdot 3$. **2.** $2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$. **3.** $2 \cdot 3 \cdot 7$. **4.** $3 \cdot 2 \cdot 11$. **5.** $2 \cdot 5 \cdot 11$. **6.** $5 \cdot 3 \cdot 7$. **7.** $2 \cdot 11 \cdot 3 \cdot 7$. **8.** $5 \cdot 3 \cdot 3$. **9.** $3 \cdot 3 \cdot 3 \cdot 7$. **10.** $5 \cdot 7 \cdot 19$. **11.** $3 \cdot 13 \cdot 11$. **12.** $5 \cdot 5 \cdot 17$. **13.** $2 \cdot 3 \cdot 3 \cdot 23$. **14.** $3 \cdot 7 \cdot 7 \cdot 23$. **15.** $23 \cdot 29$.

Article 100. — **1.** 4788. **2.** 2. **3.** 18. **4.** $5\frac{1}{3}$. **5.** $3\frac{1}{7}$. **6.** $1\frac{1}{8}$. **7.** $63\frac{7}{9}$. **8.** 12 bushels. **9.** 20 yards. **10.** 21¢. **11.** $111\frac{1}{9}$ bushels. **12.** 25 sacks.

Article 106. — **1.** 12. **2.** 21. **3.** 15. **4.** 56. **5.** 12. **6.** 20. **7.** 16. **8.** 12. **9.** 8. **10.** 2. **11.** 15. **12.** 9. **13.** 11. **14.** 63. **15.** 9.

Article 111. — **1.** 270. **2.** 36. **3.** 48. **4.** 240. **5.** 720. **6.** 72. **7.** 420. **8.** 560. **9.** 7560. **10.** 1400. **11.** 630. **12.** 192.

Article 115. — **1.** 10. **2** 14. **3.** 17. **4.** 20. **5.** 10. **6.** 6. **7.** 35. **8.** 20. **9.** 10. **10.** 3. **11.** 29. **12.** 3. **13.** 218. **14.** 93. **15.** 300. **16.** 18. **17.** 9. **18.** 40. **19.** 5. **20.** $36 + 15 + 16 + 38$. **21.** $16 \div 4$; $\frac{16}{4}$. $\frac{5+17}{11}$; $(5 + 17) \div 11$. **22.** $3 + 5 + 7$. **23.** $2 + 5 + 6$. **24.** $\$10.16 - \$4.40$. **26.** $60 - (18 + 12)$. **27.** $(18 + 5) - 8$. **28.** $(\$1.00 + \$.75 + \$2.50) - \$2.75$. **29.** $\$5.18 - (\$1.50 + \$.75)$. **30.** $4 + (3 \times 2)$. **31.** $(4 + 3) \times 2$. **32.** $4 \times 3 + 2$. **33.** $4 \times (3 + 2)$. **34.** $84 - 5 \times 6$. **35.** $(72 - 42) \times 4$. **36.** $18 \times 18 - 144$. **37.** $6 \times (11 - 6)$. **38.** $(53 + 63) \times 4$. **39.** $55¢ + (5 \times 15¢)$.

Article 132. — **1.** $\frac{2}{3}$. **2.** $\frac{2}{3}$. **3.** $\frac{1}{3}$. **4.** $\frac{9}{19}$. **5.** $\frac{19}{43}$. **6.** $\frac{5}{9}$. **7.** $\frac{1}{6}$. **8.** $\frac{9}{16}$. **9.** $\frac{107}{158}$. **10.** $\frac{1}{3}$. **11.** $\frac{20}{59}$. **12.** $\frac{31}{56}$. **13.** $\frac{1}{3}$. **14.** $\frac{1}{8}$. **15.** $\frac{7}{118}$. **16.** $\frac{11}{14}$. **17.** $\frac{130}{251}$. **18.** $\frac{7}{15}$. **19.** $\frac{3}{17}$. **20.** $\frac{2}{5}$. **21.** $\frac{1}{3}$. **22.** $\frac{1}{2}$. **23.** $\frac{147}{317}$. **24.** $\frac{1}{9}$. **25.** $\frac{1}{7}$. **26.** $\frac{11}{34}$. **27.** $\frac{1}{5}$. **28.** $\frac{11}{13}$. **29.** $\frac{2}{3}$. **30.** $\frac{1}{4}$. **31.** $\frac{1}{5}$. **32.** $\frac{4}{7}$. **33.** $\frac{46}{51}$.

Article 137. — **17.** $\frac{76}{3}$. **18.** $\frac{129}{14}$. **19.** $\frac{154}{9}$. **20.** $\frac{181}{7}$. **21.** $\frac{170}{11}$. **22.** $\frac{347}{15}$. **23.** $\frac{367}{9}$. **24.** $\frac{485}{13}$. **25.** $\frac{742}{15}$. **26.** $\frac{751}{30}$. **27.** $\frac{829}{14}$. **28.** $\frac{1077}{16}$. **29.** $\frac{2801}{29}$. **30.** $\frac{1186}{9}$. **31.** $\frac{9473}{35}$. **32.** $\frac{2225}{12}$. **33.** $\frac{3223}{11}$. **34.** $\frac{11449}{7}$. **35.** $\frac{6344}{37}$. **36.** $\frac{24773}{18}$. **37.** $\frac{11435}{48}$. **38.** $\frac{221102}{45}$. **39.** $\frac{10886}{31}$. **40.** $\frac{22967}{12}$. **41.** $\frac{38665}{19}$. **42.** $\frac{12385}{13}$. **43.** $\frac{16770}{119}$. **44.** $\frac{198822}{1269}$.

Article 142. — **1.** $47\frac{3}{4}$. **2.** $15\frac{1}{25}$. **3.** $20\frac{6}{7}$. **4.** $30\frac{7}{27}$. **5.** $30\frac{7}{32}$. **6.** $21\frac{8}{37}$. **7.** $60\frac{1}{5}$. **8.** $21\frac{44}{45}$. **9.** $16\frac{18}{57}$. **10.** $134\frac{14}{25}$. **11.** $52\frac{7}{18}$. **12.** $241\frac{14}{44}$. **13.** $27\frac{15}{37}$. **14.** $248\frac{31}{39}$. **15.** $26\frac{17}{37}$. **16.** $19\frac{1}{3}$. **17.** $19\frac{19}{46}$. **20.** $74\frac{15}{67}$. **21.** $91\frac{55}{73}$. **22.** $274\frac{5}{104}$.

Art. 147. — **1.** $\frac{15, 36, 50}{60}$. **2.** $\frac{30, 36, 20}{45}$. **3.** $\frac{25, 36, 20}{40}$. **4.** $\frac{14, 12, 27}{18}$.

5. $\frac{6, 9, 10, 5}{12}$. **6.** $\frac{30, 72, 100, 105}{120}$. **7.** $\frac{360, 25, 36, 32}{40}$. **8.** $\frac{6, 9, 10, 7}{12}$.
9. $\frac{135, 108, 80, 162}{180}$. **10.** $\frac{25, 28, 30, 32}{40}$. **11.** $\frac{15, 12, 20, 14}{30}$. **12.** $\frac{48, 45, 30, 110}{120}$.

Article 154. — **1.** $1\frac{2}{7}$. **2.** 1. **3.** $1\frac{11}{18}$. **4.** $2\frac{1}{4}$. **5.** $2\frac{1}{5}$. **6.** $1\frac{9}{16}$. **7.** $1\frac{11}{26}$. **8.** $1\frac{1}{2}$. **9.** $2\frac{67}{120}$. **10.** $7\frac{179}{180}$. **11.** $2\frac{853}{1980}$. **12.** $2\frac{33}{40}$. **13.** $9\frac{17}{30}$. **14.** $4\frac{1}{8}$. **15.** $5\frac{5}{9}$. **16.** $6\frac{1}{10}$. **17.** $16\frac{67}{168}$. **18.** $4\frac{1}{3}$. **19.** $7\frac{3}{10}$. **20.** $6\frac{1}{4}$. **21.** $28\frac{13}{24}$. **22.** $23\frac{6}{7}$. **23.** $3\frac{3}{4}$. **24.** $66\frac{19}{24}$. **25.** $47\frac{27}{56}$. **26.** $126\frac{8}{15}$ mi. **27.** $208\frac{3}{8}$ bu. **28.** $240\frac{20}{21}$. **29.** $259\frac{7}{24}$ yd. **30.** $10\frac{1}{72}$ yd. **31.** $25\frac{33}{40}$. **32.** $241\frac{104}{105}$ A.

Article 158. — **1.** $\frac{5}{12}$. **2.** $\frac{1}{6}$. **3.** $\frac{3}{8}$. **4.** $5\frac{2}{3}$. **5.** $2\frac{3}{4}$. **6.** $12\frac{1}{12}$. **7.** $4\frac{1}{12}$. **8.** $9\frac{2}{7}$. **9.** $6\frac{3}{4}$. **10.** $4\frac{1}{4}$. **11.** $\frac{13}{18}$. **12.** $\frac{5}{12}$. **13.** $\frac{3}{70}$. **14.** $\frac{62}{105}$. **15.** $\frac{123}{232}$. **16.** $\frac{19}{80} \cdot \frac{29}{220}$. **17.** $1\frac{2}{3}$. **18.** $10\frac{17}{60}$. **19.** $\frac{49}{90}$.

Article 162. — **1.** $6\frac{1}{2}$. **2.** $3\frac{1}{3}$. **3.** $2\frac{4}{5}$. **4.** $3\frac{1}{2}$. **5.** $6\frac{3}{4}$. **6.** $6\frac{14}{15}$. **7.** $5\frac{2}{3}$. **8.** $4\frac{1}{2}$. **9.** $5\frac{3}{5}$. **10.** $7\frac{3}{7}$. **11.** 1. **12.** $4\frac{11}{16}$. **13.** $2\frac{4}{5}$. **14.** 11. **15.** $4\frac{2}{5}$. **16.** 15. **17.** $11\frac{5}{11}$. **18.** $3\frac{5}{19}$. **26.** 168. **27.** 1139. **28.** $1634\frac{3}{8}$. **29.** $1064\frac{6}{17}$. **30.** 1616. **31.** $26\frac{4}{15}$. **32.** $317\frac{1}{4}$. **33.** $498\frac{1}{3}$. **34.** $1828\frac{1}{8}$. **35.** 234. **36.** 7546.

Article 165. — **1.** $\frac{35}{48}$. **2.** $\frac{3}{5}$. **3.** $\frac{9}{16}$. **4.** $\frac{1}{2}$. **5.** $\frac{1}{2}$. **6.** $\frac{9}{38}$. **7.** $\frac{1}{4}$. **8.** $\frac{3}{5}$. **9.** $\frac{2}{21}$. **10.** $\frac{5}{24}$. **11.** $\frac{16}{35}$. **12.** 15. **13.** 322. **14.** $17\frac{1}{2}$. **15.** $2\frac{4}{5}$. **16.** $1\frac{1}{4}$. **17.** $9\frac{5}{8}$. **18.** 396. **19.** 32. **20.** 36. **21.** $9\frac{3}{5}$. **22.** 205. **23.** 309. **24.** 72. **25.** 323. **26.** 750. **27.** 1818. **28.** $\frac{2}{9} \cdot 2\frac{11}{35}$. **29.** $\frac{27}{40} \cdot \frac{10}{24}$. **30.** $35\frac{21}{32}$. **31.** $115\frac{83}{160}$. **32.** $30\frac{15}{32}$. **33.** Elder, $74\frac{47}{80}$; younger, $49\frac{29}{40}$. **34.** $15\frac{15}{28}$ days.

Article 169. — **1.** $\frac{3}{16}$. **2.** $\frac{1}{8}$. **3.** $\frac{4}{25}$. **4.** $\frac{3}{119}$. **5.** $\frac{15}{32}$. **6.** $\frac{3}{14}$. **7.** $\frac{13}{36}$. **8.** $\frac{11}{75}$. **9.** $\frac{1}{18}$. **10.** $\frac{1}{6}$. **11.** $\frac{5}{6}$. **12.** $\frac{1}{8}$. **13.** $\frac{25}{32}$. **14.** $\frac{5}{42}$. **15.** $\frac{1}{8}$. **17.** $6\frac{41}{49}$. **18.** $76\frac{23}{25}$. **19.** $359\frac{7}{8}$. **20.** $77\frac{4}{5}$. **21.** $3\frac{9}{25}$. **22.** $4\frac{19}{49}$. **23.** $66\frac{3}{28}$. **24.** $1637\frac{26}{39}$. **25.** $\$\frac{71}{128}$. **26.** $11,476\frac{1}{25}$.

Article 173. — **4.** $1\frac{1}{6}$. **5.** $1\frac{1}{10}$. **6.** $\frac{10}{27}$. **7.** $\frac{8}{9}$. **8.** $1\frac{1}{3}$. **9.** $3\frac{3}{4}$. **10.** $11\frac{2}{3}$. **11.** $\frac{12}{253}$. **12.** $\frac{1}{6}$. **13.** $\frac{26}{165}$. **14.** $2\frac{1}{2}$. **15.** $11\frac{3}{7}$. **16.** 12. **17.** $\frac{1}{15}$. **18.** $\frac{2}{21}$. **19.** $\frac{1}{6}$. **20.** $4\frac{3}{4}$. **21.** 3. **22.** $2\frac{6}{11}$. **23.** $\frac{511}{576}$. **25.** $9\frac{9}{20}$. **26.** $\frac{4}{15}$. **27.** $2\frac{1}{2}$. **28.** $\frac{1}{15}$. **30.** $79\frac{1}{9}$. **31.** $88\frac{32}{41}$. **32.** $75\frac{18}{21}$. **33.** $28\frac{4}{21}$. **34.** 12,451$\frac{7}{8}$. **35.** 8001$\frac{3}{11}$. **36.** $8279\frac{5}{37}$. **37.** $3822\frac{10}{21}$. **38.** $574\frac{2}{11}$ rods. **39.** 10 days. **40.** $\$4\frac{3}{5}$. **41.** $\$3\frac{1}{4}$. **42.** 44 barrels.

Article 176. — **4.** 27. **5.** $23\frac{1}{8}$. **6.** $3\frac{15}{16}$. **7.** $\frac{17}{288}$. **8.** $1\frac{11}{84}$. **9.** $1\frac{5}{9}$. **10.** 5. **11.** 32. **12.** $\frac{63}{160}$. **13.** $4\frac{4}{21}$. **14.** $\$120\frac{1}{4}$. **15.** 14 rods. **16.** 14 tons.

Article 177. — **6.** 9. **7.** 10. **8.** 16. **9.** 9. **10.** 12. **11.** 25. **12.** 20. **13.** 6. **14.** 15. **15.** 21. **16.** 30. **17.** 40. **18.** $\frac{1}{8}$. **19.** $\frac{2}{3}$. **20.** $\frac{2}{5}$. **21.** $\frac{3}{5}$. **22.** $\frac{4}{9}$. **23.** $\frac{3}{5}$. **24.** 49. **25.** $\frac{2}{7}$. **26.** 39. **27.** $\frac{13}{24}$. **28.** 28. **29.** 209. **30.** $\frac{1}{5}$. **31.** 20. **32.** 42 marbles. **33.** 56 marbles. **34.** $\frac{3}{4}$. **35.** 70 acres. **36.** $\frac{1}{4}$ left; $\frac{3}{4}$ spent. **37.** 280 sheep.

Article 178. — **14.** $\$81\frac{1}{2}$.

Article 180. — **12.** 24 hats. **13.** 20 pounds. **14.** 24 doz. **15.** 24 pounds. **16.** 30 yd. **17.** 18 knives. **18.** $3. **19.** 112.

Article 182. — **1.** $\$185\frac{1}{4}$. **2.** $\$34\frac{1}{4}$. **3.** $32\frac{1}{8}$. **4.** Difference $\frac{1}{6}$ greater. **5.** $\$90\frac{2}{3}$. **6.** $\$200$. **7.** $380\frac{1}{5}$ barrels. **8.** Diminish $\frac{1}{24}$; $\frac{1}{40}$. **9.** $2\frac{11}{30}$. **10.** $10\frac{6}{17}$. **11.** $\$3\frac{1}{8}$. **12.** $\$.65\frac{5}{8}$. **13.** $13\frac{5}{8}$ yd. **14.** $\frac{3}{8}$; $\frac{3}{8}$. **15.** $\frac{53}{78}$; $\frac{25}{78}$. **16.** 100 sheep; 50 sheep. **17.** $\frac{8}{15}$. **18.** 125 sheep. **19.** $\$.08\frac{7}{8}$. **20.** $\$2$. **21.** $51\frac{7}{8}$ lb. **22.** $\$67\frac{1}{8}$. **23.** $\frac{1}{3}$. **24.** 120 days. **25.** 50 pupils. **26.** $\$1.50$. **27.** $\$\frac{1}{8}$. **28.** $8\frac{3}{14}$.

Article 188. — **1.** .25. **2.** .85. **3.** .326. **4.** .6. **5.** .16. **6.** .384. **7.** .49.

8. .5. **9.** .06. **10.** .015. **11.** .025. **12.** .11. **13.** .09. **14.** .500. **15.** .50. **16.** .9. **17.** .1. **18.** 5.40. **19.** 8.2. **20.** 64.683.

Article 189. — **21.** $\frac{36}{100}$. **22.** $\frac{7}{10}$. **23.** $\frac{125}{1000}$. **24.** $12\frac{2}{10}$. **25.** $6\frac{25}{100}$. **26.** $\frac{485}{1000}$. **27.** $\frac{16}{1000}$. **28.** $\frac{16}{100}$. **29.** $\frac{6}{100}$. **30.** $\frac{6}{10}$. **31.** $5\frac{6}{10}$. **32.** $5\frac{6}{100}$. **33.** $5\frac{6}{1000}$. **34.** $5\frac{600}{1000}$. **35.** $5\frac{60}{1000}$.

Article 190. — **36.** $\frac{4}{10}$; .4. **37.** $\frac{75}{100}$; .75. **38.** $\frac{125}{1000}$; .125. **39.** $16\frac{48}{100}$; 16.48. **40.** $12\frac{4}{10}$; 12.4. **41.** $\frac{6}{10}$; .6. **42.** $\frac{6}{100}$; .06. **43.** $\frac{6}{1000}$; .006. **46.** .282. **47.** .56. **48.** .7. **49.** .600.

Article 194. — **1.** Three hundred sixty-eight thousandths.

2. Eight hundred ninety-four thousandths.

3. Five thousand three hundred twenty-eight ten-thousandths.

4. Two thousand fifty-three ten-thousandths.

5. Twenty-five and six hundred twenty-three thousandths.

6. Seven and sixty-three ten-thousandths.

7. Twenty-eight and three thousand five ten-thousandths.

8. Twenty-eight thousand nine hundred sixty-two hundred-thousandths.

9. Fifteen and sixty thousand five hundred thirty-four hundred-thousandths. **10.** Thirty-seven and five hundred thirty-seven hundred-thousandths. **11.** Twenty-five and two-hundred three thousand six hundred two millionths. **12.** Thirty-eight and six millionths. **13.** Four million nine hundred eighty-three thousand six hundred ninety-five ten-millionths. **14.** Four and ninety-eight thousand three hundred sixty-nine hundred-thousandths. **15.** Forty-nine and eight thousand three hundred sixty-nine ten-thousandths. **16.** Four hundred ninety-eight and three hundred sixty-nine thousandths. **17.** Four hundred millionths. **18.** Four ten-thousandths.

Article 196. — **1.** .8. **2.** .29. **3.** 16.284. **4.** .4584. **5.** .25. **6.** .025. **7.** .0025. **8.** .00025. **9.** .000025. **10.** 1650.464. **11.** 1001.00036. **12.** 16.006. **13.** .000784. **14.** .00012. **15.** .0075.

Article 200. — **1.** .5000; .3650; .4689. **2.** .18963; .50000; 7.84000; .16005. **3.** .28000; 3.50000; .00005; .25600. **4.** .50000; .05000; .00500; .00050; .00005. **5.** .046300; .030000; .100000; .100010. **6.** .380; 1.160; .400; 78.592.

Article 201. — **2.** $\frac{1}{4}$. **3.** $\frac{7}{20}$. **4.** $\frac{3}{4}$. **5.** $\frac{16}{25}$. **6.** $\frac{13}{25}$. **7.** $\frac{19}{50}$. **8.** $\frac{1}{8}$. **9.** $\frac{7}{8}$. **10.** $\frac{3}{8}$. **11.** $\frac{91}{200}$. **12.** $\frac{1}{40}$. **13.** $\frac{561}{1000}$. **14.** $\frac{46}{125}$. **15.** $16\frac{3}{4}$. **16.** $\frac{1}{800}$. **17.** $\frac{27}{500}$. **18.** $\frac{1}{40}$. **19.** $\frac{11}{800}$. **20.** $\frac{3}{8}$.

Article 202. — **21.** $\frac{1}{8}$. **22.** $\frac{5}{8}$. **23.** $\frac{1}{16}$. **24.** $\frac{29}{160}$. **25.** $\frac{3}{80}$. **26.** $\frac{129}{800}$. **27.** $\frac{7}{8}$. **28.** $\frac{2}{3}$. **29.** $\frac{59}{160}$. **30.** $16\frac{1}{4}$; $21\frac{3}{8}$; $34\frac{3}{4}$.

Article 209. — **1.** .8. **2.** .75. **3.** .44$\frac{4}{9}$. **4.** .875. **5.** .15. **6.** .4. **7.** .6. **8.** .55$\frac{5}{9}$. **9.** .46$\frac{2}{3}$. **10.** $1\frac{1}{2}$. **11.** .30. **12.** .5625. **13.** .125. **14.** 2.24. **15.** 3.0625. **16.** .375. **17.** .0375. **18.** .32. **19.** .75. **20.** 2.09375.

Article 210. — **2.** 25.4162. **3.** 6.788. **4.** 20.2603. **5.** 194.425. **6.** 161.1095. **7.** 25401.3123. **8.** 64.4266. **9.** 120.9384. **10.** 38062.881. **11.** 25.2477. **12.** 2018.7147.

Article 211. — **1.** 19.8. **2.** 1.535. **3.** 2.85. **4.** 2.455. **5.** 6.85. **6.** .996. **7.** 10.9. **8.** 2.206. **9.** 254.844. **10.** 14.185. **11.** 3.727. **12.** 1.11. **13.** 499.95. **14.** 10.3844. **15.** 13.775. **16.** .099. **17.** Same. **18.** 999.999. **19.** 499.95.

Article 212. — **1.** .027. **2.** .2001. **3.** .15. **4.** .28. **5.** .008. **6.** 28.8075. **7.** 286.3. **8.** 17.29. **9.** 1. **10.** .05028. **11.** 26.586. **12.** 7.1. **13.** 41. **14.** 1111.

Article 215. — **10.** 5.1. **11.** 4.06+. **12.** 331. **13.** 10.1. **14.** 111.1.

15. 11110. **16.** 17.5. **17.** 70. **18.** 626.66⅔. **19.** 6266.66⅔. **20.** 62.66⅔. **21.** 62.66⅔. **22.** 375. **23.** 155.5. **24.** 10. **25.** .1. **26.** .01. **27.** 549.77. **28.** 11.94.

Article 217. — **7.** .875; .71⅜; .8; 16.75; 25.875. **8.** $\frac{7}{25}$; $\frac{13}{50}$; $\frac{3}{8}$; 15⅛. **9.** $60. **10.** 20 bbl. **13.** 25 pk. **14.** $90.657.

Article 223. — **1.** $88.90. **2.** $410.18½. **3.** $6.39. **4.** $143. **5.** 102.15. **7.** $417.175.

Article 232. — **1.** 10,368; 13,825. **2.** 5; 6½. **3.** 216; 864. **4.** ⅓; ⅔. **5.** ⅕; ⅓. **6.** $250.

Article 237. — Written: **1.** 400 oz. **2.** 15,000 lb. **3.** 258 bullets. **4.** 80 ¢. **5.** 85¼ oz. **6.** 5 oz. **7.** 30 bags. **8.** 44 oz. **9.** $3.72. **10.** 292 oz. **11.** $\frac{1}{20}$.

Article 241. — Written: **1.** 5⅓ hours. **2.** 744 hours. **3.** Winter. **4.** Yes.

Article 242. — **12.** 5½. **13.** $1⅓. **14.** $215.20. **15.** $9.60.

Article 245. — **4.** 645 in. **5.** 773 ft. **6.** 204,978 in. **7.** 81,701 sq. yd. **8.** 25,679,196 sq. in. **9.** 790,596 cu. in. **10.** 127 pt. **11.** 507 pt. **12.** 528 qt. **13.** 146,794 gr. **14.** 73,774 oz. **15.** $24. **16.** 225,932 in. **17.** 35,640 ft. **18.** 19,138,464 sq. in. **19.** 762,051 cu. in. **20.** 69,056 oz. **21.** 21,972 gr. **22.** 1947 gi. **23.** 24,000 sheets. **24.** 39,180 min.

Article 247. — **4.** 3 mi. 4 fur. 20 rd. 5 yd. 2 ft. 8 in. **5.** 6 mi. 20 rd. **6.** 17 A. 67 sq. rd. 3 sq. yd. 7 sq. ft. 72 sq. in. **7.** 16 cu. yd. 9 cu. ft. 3 cu. in. **8.** 2 tons 316 lb. **9.** 3 lb. 7 oz. 18 pwt. 4 gr. **10.** 60 gal. 3 qt. 3 gi. **11.** 5 bales. **12.** 3 wk. 6 da. 5 hr. **13.** 560 bu. 1 qt. **14.** 5 lb. 7 oz. 16 pwt. **15.** 7 lb. 3 oz. 4 dr. 1 sc. 12 gr. **16.** 17 tons 1682 lb. **17.** 43 G. gro. 10 gro. 7 doz. 4. **18.** 21 bu. 2 pk. 4 qt. **19.** 3 sq. rd. 18 sq. yd. 6 sq. ft. 27 sq. in. **20.** 2 mi. 192 rd. 2 yd. **21.** 60 rd. 4 yd.

Article 248. — **1.** 63. **2.** 244 gi. **3.** 34½ gal. **4.** $2.60. **5.** 4000 oz. **6.** 35,000 lb. **7.** 1032 bullets. **8.** 30,320 dr. **9.** 254 pt. **10.** 423 qt. **11.** $41.12. **12.** 127 bu. 4 qt. **13.** $19.50. **14.** $1.26. **15.** 4 gal. **16.** 72 qt. **17.** $4.20. **18.** 255 qt. **19.** 24 qt. **20.** 9 bu.

Article 249. — **4.** 21 bu. 3 pk. 7 qt. **5.** 15 bbl. 25 gal. 3 qt. **6.** 5 yd. 2 ft. 9 in. **7.** 13 bbl. 15 gal. 2 qt. 1 pt. **8.** 10 bu. 1 pk. 2 qt. **9.** 14 lb. 3 oz. 15 dr. **10.** 3 bbl. 28 gal. 1 pt.

Article 250. — **2.** 5 A. 104 sq. rd. 2 sq. ft. **3.** 54 da. 21 hr. 29 min. 48 sec. **4.** 3 hr. 42 min. 40 sec. **5.** 14 T. 19 cwt. 49 lb. 14 oz. **6.** 11 lb. 4 oz. 1 dr. **7.** 4 yd. 1 ft. 2 in. **8.** 10 da. 22 hr. 18 min. **9.** 9 gal. 1 pt. 3 gi. **10.** 2 dollars 1 dime 4 cents 2 mills. **11.** 2 lb. 2 oz. 12 dr. **12.** 2 gal. 3 qt. 1 pt. **13.** 11 ft. 10 in.

Article 251. — **3.** 155 yr. 6 mo. 23 da. **4.** 3 yr. 11 mo. 25 da. **5.** 67 yr. 9 mo. 22 da. **7.** 138 d. **8.** 60 da. **9.** 213 da. **10.** 23 da. **11.** 127 da. **12.** 37 da. **13.** 102 da. **14.** 120 da. **15.** 332 da.

Article 252. — **2.** 64 gal. 1 qt. 2 gi. **3.** 101 bu. 1 pk. 4 qt. **4.** 6 T. 13 cwt. 25 lb. **5.** 33 oz. 11 pwt. 6 gr. **6.** 178 gal. 1 pt. **7.** 10 lb. **8.** 352 bu. 3 pk. 4 qt. **9.** 58 hr. 40 min. 48 sec.

Article 253. — **2.** 10 bu. 3 pk. 7 qt. **3.** 2 bu. 2 pk. 2 qt. **4.** 3 qt. 1 pt. 3⅓ gi. **5.** 1 bu. 1 pk. 1 qt. 1¼ pt. **6.** 15 packages. **7.** 12 T. 16 cwt. 90 lb. 15 oz. **8** 35 packages. **9.** 28 gal. 1 qt. 1 pt. **10.** 21 gal. 1⅛ pt. **11.** 31 gal. 2 qt. **12.** 1 T. 640 lb. **13.** 106 oz. 12 pwt. **14.** 2 wk. 4 da. 13 hr. 122 min.

Article 256. — **1.** 22 sq. ft.; 3168 sq. in. **2.** 666 sq. ft. **3.** $66.00. **4.** 15 ft. **5.** 165 sq. ft. **6.** 80 sq. ft.; 11,520. **7.** 32 rd. **8.** 60 ft. **9.** 160 A.

Article 257. — **2.** 64⅔ sq. rd. **3.** $16.16⅔. **4.** $40.83⅓. **5.** 311⅑ sq. yd. **6.** $37.80.

Article 258. — **3.** 80 yd. ; 84 yd. **4.** $46.66\frac{2}{3}$. **5.** $48.22\frac{11}{12}$. **6.** 40 yd. **8.** $6475.00. **9.** 825 sq. yd. **10.** $58.33\frac{1}{3}$. **11.** 2000. **12.** 100 rd.

Article 259. — **1.** 96 sq. yd. ; 4 sq. yd. ; 24 rolls ; $81\frac{2}{3}$ sq. yd. ; $20\frac{5}{12}$ rolls ; $6.80\frac{5}{8}$; $11.00. **2.** $35.79\frac{3}{8}$. **3.** $28\frac{3}{16}$ rolls.

Article 260. — **1.** 16 bd. ft. **2.** $3.60. **3.** $16\frac{1}{8}$ bd. ft. **4.** 288 bd. ft.

Article 262. — **1.** 288 cu. in. **2.** 120 cu. ft. **3.** 384 cu. in. **4.** 8 ft. thick. **5.** 240 cu. in. **6.** 1728 blocks. **7.** $301.50. **8.** 19,440. **9.** 210 cu. in. **10.** 30 pupils. **11.** 288 cu. yd. **12.** 256 books. **13.** $13\frac{1}{2}$ cu. ft. **14.** 4320 bricks. **15.** 13,824 cu. in. **16.** 10,368 cubes. **17.** $404\frac{4}{9}$ loads. **18.** $231\frac{1}{9}$ loads. **19.** 24,300 bricks. **20.** 15 ft. wide. **21.** 10 ft. high.

Article 264. — **1.** $2\frac{1}{4}$ cd. **2.** 10 cd. **3.** $79\frac{11}{16}$ cd. **4.** $2250. **5.** 32 ft. **6.** $3\frac{3}{8}$ cd. **7.** $40\frac{1}{2}$ cd.

Article 265. — **1.** $7\frac{1}{2}$ gal. **2.** 1125 gal. **3.** $35\frac{5}{7}$ bbl. **4.** $41\frac{1}{4}$ bbl. **5.** $17\frac{9}{28}$ bbl. **6.** 1125 gal. **7.** 200 cu. ft.

Article 266. — **9.** 96 bu. **10.** 120 bu. **11.** $187\frac{1}{2}$ cu. ft.

Article 267. — **4.** 4. **5.** 10. **6.** 18. **7.** 6 sheep. **8.** 150 men. **9.** 6 bu. **10.** 64. **11.** 150. **12.** 2100. **13.** 400 sheep. **14.** 400 pupils. **15.** 500 horses. **16.** 25%. **17.** 5%. **18.** 10%. **19.** 25%. **20.** 25%. **21.** 50%. **22.** 2 apples. **23.** $150. **24.** 25%. **25.** 120 bu. **26.** 800 men. **27.** 50%.

Article 275. — **1.** 40¢. **2.** 20 sheep. **3.** 100 A. **4.** $20. **6.** $400. **7.** $400. **8.** $4. **9.** 100 sheep. **10.** $1600. **11.** $4000. **12.** $500. **13.** $50. **14.** $1500. **15.** 10%. **16.** 20%. **18.** 25% ; 75%. **19.** 80%. **20.** 25%. **21.** 25% ; 75%.

Article 277. — **6.** $10. **7.** $125. **8.** $6 ; $9 ; $15 ; 19\frac{1}{2}$.

Article 278. — **10.** $46.146. **11.** $97.386. **12.** $100.20. **13.** $164.448. **14.** $67.066. **15.** $123.976. **16.** $42.757. **17.** $79.31. **18.** $347.075. **19.** $387.915. **20.** $363.698. **21.** $373.99. **22.** $541.35.

Article 279. — **2.** $1.198. **3.** $2.716. **4.** $14.001. **5.** $5.388. **6.** $9.07. **7.** $13.498. **8.** $93.205. **9.** $8.775.

Article 280. — **1.** Five thousand seven. **2.** 14017. **3.** 211. **4.** CCXLX. **5.** 11899. **8.** $673\frac{33}{53}$. **10.** 16.

Article 281. — **2.** $\frac{32}{40}$. **3.** $\frac{7}{18}$. **4.** 120. **6.** $11\frac{1}{6}$. **7.** $7\frac{3}{4}$. **8.** $\frac{2}{5}$. **9.** $10\frac{11}{40}$; $21\frac{7}{18}$; $9\frac{21}{23}$. **10.** $70\frac{1}{6}$.

Article 282. — **4.** $\frac{1}{20}$. **5.** 7. **6.** $\frac{107}{117}$. **7.** $\frac{37}{128}$. **8.** $\frac{1}{6}$. **9.** $300. **10.** 70 sheep.

Article 283. — **1.** 2 tenths ; 2 tens. **2.** Same. **3.** 999.999. **4.** 7.10. **5.** 1. **6.** 111.1 ; 10 ; .1. **7.** .6 ; .25 ; $.33\frac{1}{3}$; .625. **8.** $\frac{54}{10000000} = \frac{27}{5000000}$. **9.** 34.0003406. **10.** 15.0005.

Article 284. — **1.** $28.72. **3.** $\frac{5}{8}$; $\frac{5}{6}$. **4.** 23.0310. **5.** .000001. **6.** 8.25. **7.** $60.525. **8.** $36920.

Article 285. — **1.** $13.333. **2.** 27 yd. **3.** 600 ; 60 ; 6 units. **4.** $\frac{2}{3}$; $\frac{1}{12}$ greater. **5.** $\frac{1}{4}$. **6.** $5.61. **7.** 200 bu. **8.** 5438.857. **9.** 170 marbles. **10.** 800.

Article 286. — **1.** 12 bu. **2.** 3 mi. 23 rd. 1 yd. **3.** 405 rd. **5.** 2 qt. **6.** 18 bu. **7.** 5 qt. **8.** 3 tons. **9.** $3.60. **10.** $26.10.

Article 287. — **1.** 48 yd. 2 ft. 7 in. **2.** $1.60. **3.** $\frac{3}{5}$. **4.** 18 ft. **5.** 40 yd. **6.** 33264 sq. in. **7.** 860 A. **8.** 73 hhd. 18 gal. 2 qt. **9.** 5280 ft.

Article 288. — **3.** 16 yd. **4.** $27. **5.** $\$\frac{3}{4}$. **6.** $1.05. **7.** $16.20. **8.** 198 sq. ft. **9.** $1122\frac{3}{10}$ using $7\frac{1}{2}$; $1119\frac{3}{11}$ using 231. **10.** 96.427+ bu.

Article 289. — **1.** 9 ft. **2.** 8 farms. **3.** 500 lb. **4.** $\frac{3}{16}$. **5.** 7 cu. yd. 24 cu. ft. 1362 cu. in. **6.** 10 cd. **7.** $20\frac{55}{256}$ cd. **8.** 173.569+ bu. **9.** $38. **10.** 160 rd.